New Liquidity Risk in a Multi-period Investment Horizon

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We analytically find a new source of illiquidity risk in Merton (1973)'s intertemporal CAPM, which is the covariance between stock returns and state variables, expanding Acharya and Pedersen (2005). Using two stock market risk factors and two bond market risk factors as state variables to capture a shift in the investment opportunity set, we find that the new illiquidity risk is priced. After controlling the new illiquidity risk, Acharya and Pedersen (2005)'s two illiquidity risks are not priced anymore.

JEL classification: G11, G12

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1. Introduction

Liquidity in the stock market means how quick an investor's position can be liquidated without price drop. During the subprime mortgage crisis or in prior sharply illiquid market situations, many stock market participants perceived liquidity as an important issue and possibly as undiversifiable risk. We analytically find a new source of illiquidity risk in Merton (1973)'s intertemporal capital asset pricing model, besides three sources of illiquidity risks (Acharya and Pedersen (2005)). And empirically we find that the new source of illiquidity risk is priced, using equity factors and bond factors as state variables, with conditionally estimated covariance risks of multivariate GARCH model. We argue that one more type of liquidity risk can be added to the asset pricing literature, which can be referred to in fund performance evaluation, portfolio management, etc.

Literatures related to stock liquidity have evolved from the relation between liquidity level and return to liquidity risk identification in asset pricing framework. In early studies such as Amihud and Mendelson (1986), Brennan and Subrahmanyam (1996), and Amihud (2002), the main issue is the impact of liquidity level on asset prices. And the liquidity research moved on to identify liquidity risks. The higher the level of illiquidity is, the more expensive trading cost is, and thus the greater stock premium is being paid to attract investors. Also, stocks could be exposed to undiversifiable liquidity risks, requiring higher premium for the risks. So far, there have been identified three sources of illiquidity risk. Acharya and Pedersen (2005) develop an integrated model, incorporating liquidity level and three sources of liquidity risks into the CAPM. The first source of illiquidity risk is commonality in liquidity. Chordia, Roll, and Subrahmanyam (2000) investigate commonality in liquidity, which means the comovement of an individual stock's liquidity with market liquidity. Similar to this study, Hasbrouck and Seppi (2001), Huberman and

Halka (2001), Karolyi, Lee, and van Dijk (2011), and Brockman, Chung and Perignon (2009) also investigate and expand understanding on commonality in liquidity. The second source of illiquidity risk is the covariance between an individual stock's return and market illiquidity. Pastor and Stambaugh (2003), Liu (2006), Watanabe and Watanabe (2007), and Korajczyk and Sadka (2008) investigate whether this source of risk is being priced. The last source of illiquidity risk is the covariance between an individual stock's illiquidity and market return. Acharya and Pedersen (2005) investigate this source of illiquidity risk and they argue that this type of illiquidity risk is the most important.

In this study, we look into a new source of illiquidity risk, using Merton (1973)'s intertemporal CAPM. We identify the covariance between a individual stock illiquidity and state variables as new source of illiquidity risk. We select four proxies for state variables, which are the most promising variables believed to reflect a shift in the investment opportunity set. There are two categories of state variable proxies: equity market factors, including smb and hml, and bond market factors, including term spread and default spread. Apart from Acharya and Pedersen (2005), we test a conditional version of liquidity-adjusted asset pricing model, using multivariate GARCH model. Among many empirically feasible multivariate GARCH models, we follow the specification of Engle (2002) with a relatively small number of unknown parameters. We find empirical evidence that the new source of illiquidity risk is priced, in the covariance between an individual stock's relative illiquidity cost and state variables.

In the next section, we derive a new source of liquidity risk from Merton (1973)'s Intertemporal CAPM. In section 3, we explain the conditional liquidity risk estimation in multivariate GARCH framework. In section 4, we introduce data and illiquidity measures used in this paper. Then, we turn to empirical results in section 5.

2. New source of liquidity risk in Intertemporal CAPM framework

Following Merton (1973)'s Intertemporal CAPM, we assume investors maximize a time-separable expected utility function in a continuous-time environment.

$$J(W,x,t) = \max_{C_s,\{\omega_{i,s}\},\forall s,i} E_t\left[\int_t^T U(C_s,s)\,ds\right] \tag{1}$$

where C_t is the induvidual's rate of consumption per unit time at time *t*. W is defined as the value of the individual's wealth portfolio. Utility function $U(C_t,t)$ is assumed to be strictly increasing and concave in C_t.

And we have the following constraints to maximize the objective function.

$$dW = \left[\sum_{i=1}^{n} \omega_i\,(dS_i/S_i - dG_i) + \left(1 - \sum_{i=1}^{n} \omega_i\right) rdt\right] W - Cdt \tag{2}$$

$$\frac{dS_i}{S_i} = \mu_i(x,t)dt + \sigma_i(x,t)dz_i \tag{3}$$

$$dG_i = \delta_i(x,t)dt + \lambda_i(x,t)dy_i \tag{4}$$

$$dx = a(x,t)dt + b(x,t)d\zeta \tag{5}$$

Eq. (2) describes the dynamics of wealth. r is the risk-free asset return and ω_i is the proportion of total wealth allocated to risky asset *i*. Eq. (3) shows risky asset *i*'s instantaneous return where $S_{i,t}$ is the price of the *i*th risky asset at time *t*. Return distribution of risky asset *i*'s depends on state variable x. In this framework, investors face changes in the investment opportunities set. dG_i and dx describe instantaneous changes of relative illiquidity cost of risky asset *i* and state variable process, respectively. dz_i, dy_i and $d\zeta$ follow pure Brownian motion process. Therefore, each individual maximizes Eq. (1) subject to Eq. (2), choosing control variables, C_s and $\omega_{i,s}$, for time s from *t* to T.

After applying Bellman's Principle of Optimality,

$$J(\mathrm{W_t}, x_\mathrm{t}, \mathrm{t}) = \max_{C_s, \{\omega_{i,s}\}, \forall s, i} E_t \left[\int_t^{t+\Delta t} U(C_s, s)\, ds + \max_{C_s, \{\omega_{i,s}\}, \forall s, i} E_t \left[\int_{t+\Delta t}^{T} U(C_s, s)\, ds \right] \right]$$

$$= \max_{C_s, \{\omega_{i,s}\}, \forall s, i} E_t \left[\int_t^{t+\Delta t} U(C_s, s)\, ds + J(W_{t+\Delta t}, x_{t+\Delta t}, t + \Delta t) \right] \quad (6)$$

Taylor series at $\mathrm{W_t}, x_\mathrm{t}$ and t give the following equation,

$$J(\mathrm{W_t}, x_\mathrm{t}, \mathrm{t}) = \max_{C_s, \{\omega_{i,s}\}, \forall s, i} E_t [U(C_t)\Delta t + J(W_t, x_t, t) + J_W \Delta W + J_x \Delta x + J_t \Delta t + \tfrac{1}{2} J_{WW} (\Delta W)^2 +$$

$$\tfrac{1}{2} J_{xx} (\Delta x)^2 + \tfrac{1}{2} J_{tt} (\Delta t)^2 + J_{xt} (\Delta x)(\Delta t) + J_{xW} (\Delta x)(\Delta W) + J_{Wt} (\Delta W)(\Delta t) + o(\Delta t)] \quad (7)$$

Δt is a short interval of time and $o(\Delta t)$ is a higher-order term which is almost zero as $\Delta t \to 0$. Substituting Eq. (2) with Eq. (1), dividing both sides by Δt, and taking the limit as $\Delta t \to 0$ after subtracting $J(\mathrm{W_t}, x_\mathrm{t}, \mathrm{t})$ from both sides, we have

$$0 = \max_{C_s, \{\omega_{i,s}\}} E_t \left[U(C_t) + J_t + \left[\sum_{i=1}^{n} \omega_i (\mu_i - \delta_i - r) W + (rW - C) \right] J_W + a J_x + \tfrac{1}{2} W^2 \left[\sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij} \omega_i \omega_j \right] J_{WW} \right.$$

$$+ \tfrac{1}{2} W^2 \left[\sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_{ij} \omega_i \omega_j \right] J_{WW} - \tfrac{1}{2} W^2 \left[\sum_{i=1}^{n} \sum_{j=1}^{n} \psi_{ij} \omega_i \omega_j \right] J_{WW} - \tfrac{1}{2} W^2 \left[\sum_{i=1}^{n} \sum_{j=1}^{n} \tau_{ij} \omega_i \omega_j \right] J_{WW}$$

$$\left. + \tfrac{1}{2} b^2 J_{xx} + W J_{xw} \sum_{i=1}^{n} \varphi_i \omega_i + W J_{xw} \sum_{i=1}^{n} \pi_i \omega_i \right] \quad (8)$$

Now, we have a first order condition given the concavity of $U(.)$,

$$0 = U_c(C^*, t) - J_w(W, x, t) \quad (9)$$

$$0 = (\mu_i - \delta_i - r) J_W + \left[\sum_{j=1}^{n} \sigma_{ij} \omega_\mathrm{j}^* + \sum_{j=1}^{n} \lambda_{ij} \omega_\mathrm{j}^* - \sum_{j=1}^{n} \psi_{ij} \omega_\mathrm{j}^* - \sum_{j=1}^{n} \tau_{ij} \omega_\mathrm{j}^* \right] W J_{WW} + [\varphi_i - \pi_i] J_{Wx} \quad (10)$$

where $\sigma_{ij} dt = (\sigma_i dz_i)(\sigma_j dz_j)$, $\lambda_{ij} dt = (\lambda_i dy_i)(\lambda_j dy_j)$, $\psi_{ij} dt = (\sigma_i dz_i)(\lambda_j dy_j)$, $\tau_{ij} dt = (\lambda_i dy_i)(\sigma_j dz_j)$, $\varphi_i dt = (\sigma_i dz_i)(b d\zeta)$, $\pi_i dt = (\lambda_i dy_i)(b d\zeta)$.

Next, we divide Eq. (10) of individual p's optimal choice by J_{WW}^{p}, with the superscript p denoting individual p and bold type describe n assets vector,

$$\boldsymbol{\mu} - \boldsymbol{\delta} - r\boldsymbol{e} = \gamma\boldsymbol{\sigma}\boldsymbol{B} + \gamma\boldsymbol{\lambda}\boldsymbol{B} - \gamma\boldsymbol{\psi}\boldsymbol{B} - \gamma\boldsymbol{\tau}\boldsymbol{B} + \theta\boldsymbol{\varphi} - \theta\boldsymbol{\pi} \tag{11}$$

where $\gamma \equiv -\sum_p W^p / \sum_p (J_W^p / J_{WW}^p)$, $\boldsymbol{B} \equiv \sum_p \boldsymbol{\omega}^{\boldsymbol{p}} W^p / \sum_p W^p$, $\theta \equiv -\sum_p (J_{Wx}^p / J_{WW}^p) / \sum_p (J_W^p / J_{WW}^p)$, $\boldsymbol{B}$ is the average investment in each asset across investors and the market weight in equilibrium. Hence, *i*th asset is

$$\mu_i - \delta_i - r = \gamma\sigma_{im} + \gamma\lambda_{im} - \gamma\psi_{im} - \gamma\tau_{im} + \theta\varphi_i - \theta\pi_i \tag{12}$$

Eq. (12) is a natural generalization of Acharya and Pedersen (2005) with two additional risks from changing investment opportunities dependent on state variable x in a multi-period environment. While they use gross return and relative illiquidity cost, we use net return and difference of relative illiquidity cost. For the empirical test, the econometric specification comes similar to that of Acharya and Pedersen (2005) ,

$$\begin{aligned} &E_t\left(r_{t+1}^i - r\right) = \alpha + \gamma_0 E_t(c_{t+1}^i) + \gamma_1 cov_t\left(r_{t+1}^i, r_{t+1}^m\right) + \\ &\gamma_2 cov_t\left(c_{t+1}^i, c_{t+1}^m\right) + \gamma_3 cov_t\left(r_{t+1}^i, c_{t+1}^m\right) + \gamma_4 cov_t\left(c_{t+1}^i, r_{t+1}^m\right) + \\ &\theta_1 cov_t\left(r_{t+1}^i, x_{t+1}\right) + \theta_2 cov_t\left(c_{t+1}^i, x_{t+1}\right) \end{aligned} \tag{13}$$

where r_{t+1}^i is stock i's return, c_{t+1}^i is stock i's difference of relative illiquidity cost, r_{t+1}^m is market return, and c_{t+1}^m is difference of relative market illiquidity cost. Eq. (12) states that the expected return is the sum of expected differences in relative illiquidity cost, $E_t(c_{t+1}^i)$, and six covariances. Besides the conventional market risk, $cov_t\left(r_{t+1}^i, r_{t+1}^m\right)$, and the risk related to state variable x, $cov_t\left(r_{t+1}^i, x_{t+1}\right)$, there are also three sources of liquidity risks similar to Acharya and Pedersen (2005)'s suggestion, plus one new source of liquidity risk. is the first three are as follows: through commonality in liquidity, $cov_t\left(c_{t+1}^i, c_{t+1}^m\right)$, investors require extra premium for holding a stock that becomes more illiquid in an illiquid market. For a stock with positive $cov_t\left(r_{t+1}^i, c_{t+1}^m\right)$, investors are willing to accept lower return because it has high return in a more illiquid market. For a stock with positive $cov_t\left(c_{t+1}^i, r_{t+1}^m\right)$, investor are also willing to accept lower return because it has a low trading cost in a down market. Unlike Acharya and Pedersen (2005), in a multi-period investment horizon with changing investment opportunities, a new source of liquidity risk arises, $cov_t\left(c_{t+1}^i, x_{t+1}\right)$. For the new liquidity risk, $cov_t\left(c_{t+1}^i, x_{t+1}\right)$, investors are willing to accept lower return for a stock that has a lower illiquidity trading cost in a bad state.

3. Conditional liquidity risk estimation in multivariate GARCH framework

For the cross-sectional test of Liquidity-adjusted CAPM, Acharya and Pedersen (2005) derive an unconditional version of LCAPM. In this study, we employ the multivariate generalized autoregressive conditional heteroskedasticity (GARCH) for covariance risks estimation in the conditional version test of Eq. (13). Engle and Bali (2010) use a similar approach in the ICAPM test. In detail, first, we estimate conditional covariance in the multivariate GARCH framework using Engle (2002)'s dynamic conditional correlation model. Then using estimated conditional covariance, Fama-MacBeth (1973) regression is employed to obtain the coefficients and *t-value*s. Bollerslev (1990)'s constant conditional correlation for multivariate GARCH decomposes the conditional covariance into the conditional variance and the constant correlation.

$$\boldsymbol{R}_t|\boldsymbol{F}_{t-1} \sim \boldsymbol{N}(\boldsymbol{0}, \boldsymbol{H}_t),$$

$$\boldsymbol{H}_t = \boldsymbol{D}_t \boldsymbol{\rho} \boldsymbol{D}_t, \tag{14}$$

where $\boldsymbol{R_t}$ is $N \times 1$ vector (in this study, N=2), $\boldsymbol{H_t}$ is the conditional covariance matrix at time t, $\boldsymbol{D_t}$ is a $N \times N$diagonal matrix with the conditional standard deviation, $\sqrt{\sigma_{ii,t}}$, of i asset on (i,i). The conditional variances are specified using univariate GARCH (1, 1) process,

$$\sigma_{ii,t} = \alpha_0 + \alpha_1 r_{i,t-1}^2 + \alpha_2 \sigma_{ii,t-1} \tag{15}$$

Constant conditional correlation matrix, $\boldsymbol{\rho}$, is

$$\boldsymbol{\rho} = (\rho_{ij})_{ij} \,, (\; i = j \; then \; \rho_{ii} = 1) \tag{16}$$

Therefore, conditional covariance matrix is

$$\boldsymbol{H}_t = (\rho_{ij}\sqrt{\sigma_{ii,t}}\sqrt{\sigma_{jj,t}})_{ij} \tag{17}$$

The constant correlation assumption allows a simple parameterization in multivariate GARCH in contrast to other multivariate GARCH, such as BEKK (Baba, Engle, Kraft and Kroner) model. This simplifies the estimation due to the small number of unknown parameters ($N(N$+5$)/$2).

We employ the Dynamic Conditional Correlation (DCC) model of Engle (2002) to estimate liquidity risks. The DCC model can be viewed as a generalized version of Constant Conditional Correlation model of Bollerslev (1990). The DCC model allows $\boldsymbol{\rho}$ to be time-varying.

$$\boldsymbol{H_t} = \boldsymbol{D_t}\boldsymbol{\rho_t}\boldsymbol{D_t}, \tag{18}$$

where $(\boldsymbol{\rho_t})_{i,j} = \rho_{i,j,t} = q_{i,j,t}/\sqrt{q_{i,i,t}q_{j,j,t}}$,.

$$q_{i,j,t} = \bar{\rho}_{i,j} + \alpha\big(R_{i,t-1}R_{j,t-1} - \bar{\rho}_{i,j}\big) + \; \beta\big(q_{i,j,t-1} - \bar{\rho}_{i,j}\big) \tag{19}$$

When $\alpha + \beta < 1, \alpha + \beta = 1$, Eq. (19) is mean-reverting and integrated, respectively.[1]

4. Relative illiquidity cost measure

[1]For estimation, Kevin Sheppard's UCSD Toolbox is used. (http://www.kevinsheppard.com)

We use the trading cost measure of Lesmond, Ogden, and Trzcinka (1999). This measure fits directly into Acharya and Pedersen (2005) and this study's framework using relative illiquidity cost. And Goyenko, Holden, and Trzcinka (2009) argue that this LOT y-split measure, which is employed in this study, dominate other measures. Also, this measure has the longest horizon and computationally intermediate requirements.

4.1. Lesmond, Ogden, and Trzcinka (1999)'s liquidity measure

The LOT model assumes that unobserved true return R_{it}^* of a stock *i* on day *t* is given by

$$R_{it}^* = \beta_i R_{mt} + \varepsilon_{it} \tag{23}$$

where

$$R_{it} = R_{it}^* - \alpha_{1,i} \quad if \ R_{it}^* < \alpha_{1,i} \ ,$$

$$R_{it} = 0 \qquad\qquad if \ \alpha_{1,i} < R_{it}^* < \alpha_{2,i} \ ,$$

$$R_{it} = R_{it}^* - \alpha_{2,i} \quad if \ R_{it}^* > \alpha_{2,i} \ ,$$

LOT estimate of the effective spread is defined as $\alpha_{2,i} - \alpha_{1,i}$. And Lesmond, Ogden, and Trzcinka (1999) develop the following maximum likelihood estimator:

$$L(\alpha_{1,i}, \alpha_{2,i}, \beta_i, \sigma_i | R_{it}, R_{mt}) = \prod_1 \frac{1}{\sigma_i} n \left[\frac{R_{it} + \alpha_{1,i} - \beta_i R_{mt}}{\sigma_i}\right]$$

$$\times \prod_0 \left[N\left(\frac{\alpha_{2,i} - \beta_i R_{mt}}{\sigma_i}\right) - N\left(\frac{\alpha_{1,i} - \beta_i R_{mt}}{\sigma_i}\right)\right] \times \prod_2 \frac{1}{\sigma_i} n \left[\frac{R_{it} + \alpha_{2,i} - \beta_i R_{mt}}{\sigma_i}\right] \tag{24}$$

subject to $\alpha_{1,i} \leq 0, \alpha_{2,i} \geq 0, \ \beta_i > 0, \sigma_i > 0$, where *n*(.) is standard normal density function, *N*(.) is the cumulative normal distribution. Lesmond, Ogden, and Trzcinka (1999) use three regions, which are region 0 is $\mathrm{R_{it}} = 0$, region 1 is $\mathrm{R_{it}} \neq 0$ and $\mathrm{R_{mt}} > 0$, and region 2 is $\mathrm{R_{it}} \neq 0$ and $\mathrm{R_{mt}} < 0$. But following Goyenko, Holden and Trzcinka (2009), we use an alternative region, which they call LOT Y-split, where region 0 is $\mathrm{R_{it}} = 0$, region 1 is $\mathrm{R_{it}} > 0$, and region 2 is

$R_{it} < 0$. Goyenko, Holden and Trzcinka (2009) found that LOT Y-split generates a better result than the original region with respect to correlations and mean squared prediction errors with TAQ high frequency data. Also, LOT Y-split yields a better convergence rate in numerical optimization.

5. Data

We investigate the pricing of illiquidity for US stocks traded at the NYSE, AMEX and NASDAQ for the monthly frequency of January 1965 ~ December 2011. We use stock data from the Center for Research in Security Prices (CRSP), and only common stocks are included (share code: 10, 11). At least 36 month effective observations are required to be included. Two equity factors (smb, hml) are obtained from Kenneth French's online data library. Term spread, default spread and federal funds rate are obtained from H.15 database of Board of Governors of the Federal Reserve System. We use 25 size-B/M test asset constructed following Fama and French (1993).

6. Empirical results

6.1. Correlation of estimated liquidity risks

<Insert here Table 1 and Table 2>

Table 1 describes the summary statistics of Lesmond, Ogden, and Trzcinka (1999)'s liquidity measure. Overall, 5 size portfolios show the monotonic relation between size and liquidity cost in each book-to-market portfolio. And also 5 BM portfolios show the monotonic relation between book-to-market ratio and liquidity cost in each size portfolio.

Table2 describes the correlation coefficients among the estimated liquidity risks. New liquidity risks have high correlation with Acharya and Pedersen (2005)'s three liquidity risks. So we conjecture these new estimated liquidity risks could be priced in intertemporal liquidity adjusted CAPM framework.

6.2. Analysis using mean-reverting dynamic conditional correlation of Engle (2002)

<Insert here Table 3>

Panel A in Table 3 reports time-series mean and *t-value* using Fama-MacBeth cross-sectional regression of Eq. (25) using mean-reverting dynamic conditional correlation of Engle (2002), where *t-value* is computed using Newey-West correction of standard errors for heteroscedasticity and autocorrelation,

$$E_t\left(r_{t+1}^i - r\right) = \alpha + \gamma_0 E_t(c_{t+1}^i) + \gamma_1 cov_t^{MR}\left(r_{t+1}^i, r_{t+1}^m\right) + \gamma_2 cov_t^{MR}\left(c_{t+1}^i, c_{t+1}^m\right) + \gamma_3 cov_t^{MR}\left(r_{t+1}^i, c_{t+1}^m\right) + \gamma_4 cov_t^{MR}\left(c_{t+1}^i, r_{t+1}^m\right) + \theta_1 cov_t^{MR}\left(r_{t+1}^i, x_{t+1}\right) + \theta_2 cov_t^{MR}\left(c_{t+1}^i, x_{t+1}\right) \qquad (25)$$

where γ_1 is price of market risk, γ_2, γ_3, γ_4 and θ_2 are the price of four sources of illiquidity risks, respectively. Especially, we are interested in θ_2 because it is new source of illiquidity risk. Equity factors and bond factors, which are smb, hml, term spread and default spread, are used as x_t. Panel A in Table 3 using mean-reverting dynamic conditional correlation of Engle (2002), the new source of illiquidity risk (covariance between stock *i*'s illiquidity and state variable) is priced in smb, hml, term spread, and default spread as state variable. When we use smb, hml, term spread, and default spread as state variable, we have significant *t-value*s of -2.17, -2.25, -1.92, and -3.73, respectively. Following Vassalou (2003), we regard that the lower smb, hml, term premium, and default premium become, the worse the economic situation would be in terms of future GDP growth. So we have a correct and significant sign in the new liquidity risk. In Acharya and Pedersen (2005)'s liquidity risks, we have correct, significant coefficients, except commonality liquidity risk (γ_2). But after controlling new liquidity risks, they become insignificant. We interpret these means that our new liquidity risk is a more important source of liquidity risk.

6.3. Analysis using integrated dynamic conditional correlation of Engle (2002)

Panel B in Table 3 reports time-series mean and *t-value* using Fama-MacBeth cross-sectional regression of Eq. (26) using integrated dynamic conditional correlation of Engle (2002), where *t-value* is computed using Newey-West correction of standard errors for heteroscedasticity and autocorrelation.

$$E_t\left(r_{t+1}^i - r\right) = \alpha + \gamma_0 E_t(c_{t+1}^i) + \gamma_1 cov_t^{integ}\left(r_{t+1}^i, r_{t+1}^m\right) +$$

$$\gamma_2 cov_t^{integ}\left(c_{t+1}^i, c_{t+1}^m\right) + \gamma_3 cov_t^{integ}\left(r_{t+1}^i, c_{t+1}^m\right) + \gamma_4 cov_t^{integ}\left(c_{t+1}^i, r_{t+1}^m\right) +$$

$$\theta_1 cov_t^{integ}\left(r_{t+1}^i, x_{t+1}\right) + \theta_2 cov_t^{integ}\left(c_{t+1}^i, x_{t+1}\right) \tag{26}$$

The new source of illiquidity risk (covariance between stock *i*'s illiquidity and state variable) is priced in smb and default spread as state variable. When we use smb and default spread, we have significant *t-value*s of -2.78, and -2.79, respectively. Similar to Panel A in Table 3, Acharya and Pedersen (2005)'s liquidity risks are priced except commonality liquidity risk (γ_2). But after controlling new liquidity risks, γ_4 become insignificant.

7. Conclusion

We analytically find a new source of illiquidity risk in Merton (1973)'s intertemporal CAPM, which is the covariance between stock returns and state variables, expanding Acharya and Pedersen (2005) in a single period. Using two stock market risk factors and two bond market risk factors as state variables to capture a shift in the investment opportunity set, we find that the new illiquidity risk is priced. After controlling the new illiquidity risk, Acharya and Pedersen (2005)'s two illiquidity risks are not priced anymore.

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Table 1 Summary of the Liquidity Cost Measures

This table presents liquidity costs, measured using Lesmond, Ogden, and Trzcinka's (1999). Sample period is 196501~201112.

	Lesmon, Ogden, and Trzinka (1999) illiquidity measure				
	Size 1	Size 2	Size 3	Size 4	Size 5
BM 1	0.115	0.077	0.059	0.045	0.032
BM 2	0.101	0.066	0.050	0.039	0.028
BM 3	0.095	0.059	0.045	0.035	0.027
BM 4	0.089	0.055	0.043	0.034	0.026
BM 5	0.091	0.056	0.045	0.036	0.028

Table 2 Correlation Coefficients Among the Estimated Liquidity Risks

This table presents the average ($\times 10^5$) and the correlation coefficients among the estimated illiquidity risks. Test assets are 25 size-BM portfolios. Liquidity costs are measured using Lesmond, Ogden, and Trzcinka's (1999) measure (LOT). The covariances are estimated from the two GARCH specifications: Engle's (2002) mean-reverting dynamic conditional correlation model (MR), and Engle's (2002) integrated dynamic conditional correlation model (Integ).

	Acharya-Pedersen (2005) liquidity risks			Liquidity risks related with state variables			
	$\widehat{\text{Cov}}(c^i_{t+1}, c^m_{t+1})$	$\widehat{\text{Cov}}(r^i_{t+1}, c^m_{t+1})$	$\widehat{\text{Cov}}(c^i_{t+1}, r^m_{t+1})$	$\widehat{\text{Cov}}(c^i_{t+1}, \text{SMB}_{t+1})$	$\widehat{\text{Cov}}(c^i_{t+1}, \text{HML}_{t+1})$	$\widehat{\text{Cov}}(c^i_{t+1}, \text{TERM}_{t+1})$	$\widehat{\text{Cov}}(c^i_{t+1}, \text{DEF}_{t+1})$
Panel A: Estimated using Engle's (2002) mean-reverting dynamic conditional correlation model							
Average	1.37	-4.81	-5.57	-3.86	1.02	-0.25	0.28
$\widehat{\text{Cov}}(c^i_{t+1}, c^m_{t+1})$	1	-0.74	-0.6	-0.61	0.54	-0.38	0.43
$\widehat{\text{Cov}}(r^i_{t+1}, c^m_{t+1})$	-0.74	1	0.53	0.66	-0.62	0.35	-0.33
$\widehat{\text{Cov}}(c^i_{t+1}, r^m_{t+1})$	-0.6	0.53	1	0.53	-0.34	0.48	-0.51
$\widehat{\text{Cov}}(c^i_{t+1}, \text{SMB}_{t+1})$	-0.61	0.66	0.53	1	-0.64	0.3	-0.37
$\widehat{\text{Cov}}(c^i_{t+1}, \text{HML}_{t+1})$	0.54	-0.62	-0.34	-0.64	1	-0.13	0.26
$\widehat{\text{Cov}}(c^i_{t+1}, \text{TERM}_{t+1})$	-0.38	0.35	0.48	0.3	-0.13	1	-0.44
$\widehat{\text{Cov}}(c^i_{t+1}, \text{DEF}_{t+1})$	0.43	-0.33	-0.51	-0.37	0.26	-0.44	1
Panel B: Estimated using Engle's (2002) integrated dynamic conditional correlation model							
Average	1.35	-4.77	-5.33	-3.8	1.04	-0.24	0.27
$\widehat{\text{Cov}}(c^i_{t+1}, c^m_{t+1})$	1	-0.74	-0.6	-0.58	0.56	-0.38	0.42
$\widehat{\text{Cov}}(r^i_{t+1}, c^m_{t+1})$	-0.74	1	0.56	0.66	-0.64	0.34	-0.33
$\widehat{\text{Cov}}(c^i_{t+1}, r^m_{t+1})$	-0.6	0.56	1	0.51	-0.39	0.46	-0.47
$\widehat{\text{Cov}}(c^i_{t+1}, \text{SMB}_{t+1})$	-0.58	0.66	0.51	1	-0.64	0.27	-0.29
$\widehat{\text{Cov}}(c^i_{t+1}, \text{HML}_{t+1})$	0.56	-0.64	-0.39	-0.64	1	-0.14	0.27
$\widehat{\text{Cov}}(c^i_{t+1}, \text{TERM}_{t+1})$	-0.38	0.34	0.46	0.27	-0.14	1	-0.43
$\widehat{\text{Cov}}(c^i_{t+1}, \text{DEF}_{t+1})$	0.42	-0.33	-0.47	-0.29	0.27	-0.43	1

Table 3 Cross-Sectional Regression Estimates

This table presents time-series averages of the Fama-Macbeth (1973) cross-sectional regression coefficients of the following model:

$$E_t\left(r_{t+1}^i - r_f\right) = \alpha + \gamma_0 E_t(c_{t+1}^i) + \gamma_1 \mathrm{Cov_t}\left(r_{t+1}^i, r_{t+1}^m\right) + \gamma_2 \mathrm{Cov_t}\left(c_{t+1}^i, c_{t+1}^m\right) + \gamma_3 \mathrm{Cov_t}\left(r_{t+1}^i, c_{t+1}^m\right)$$
$$+\gamma_4 \mathrm{Cov_t}\left(c_{t+1}^i, r_{t+1}^m\right) + \theta_1 \mathrm{Cov_t}\left(r_{t+1}^i, x_{t+1}\right) + \theta_2 \mathrm{Cov_t}\left(c_{t+1}^i, x_{t+1}\right),$$

where c_{t+1}^i and c_{t+1}^m are the liquidity costs of test asset i and the market portfolio, respectively, and r_{t+1}^i and r_{t+1}^m are the returns of test asset i and the market portfolio, respectively, r_f is the riskless rate of return, and x_{t+1} is the state variable, which is SMB (the size factor), HML (the book-to-market factor), TERM (term spread), or DEF (default spread). Test assets are 25 size-BM portfolios. Panel A uses the covariance terms through Engle's (2002) mean-reverting dynamic conditional correlation GARCH model. Panel B uses the covariance terms through Engle's (2002) integrated dynamic conditional correlation GARCH model. Liquidity costs are measured using Lesmond, Ogden, and Trzcinka's (1999). t-statistics are below the estimates and are calculated using Newey-West heteroscedasticity and autocorrelation corrected standard error.

	α	γ_0	γ_1	γ_2	γ_3	γ_4	$\theta_{1,\mathrm{smb}}$	$\theta_{1,\mathrm{hml}}$	$\theta_{1,\mathrm{term}}$	$\theta_{1,\mathrm{def}}$	$\theta_{2,\mathrm{smb}}$	$\theta_{2,\mathrm{hml}}$	$\theta_{2,\mathrm{term}}$	$\theta_{2,\mathrm{def}}$	$adj\mathrm{R}^2$
Panel A: Engle's (2002) mean-reverting dynamic conditional correlation multivariate GARCH model															
M0	-0.0011	-0.7583	4.8264	-856.292	-243.775	-76.6115									0.3894
	-0.32	-1.96	2.16	-3.53	-3.26	-2.9									
M1	-0.0003	0.4818	2.1831	-371.093	54.5414	-2.1105	5.7629	-0.4683	31.6807	15.3681	-109.262	-161.798	-1080.49	-1254.3	0.5744
	-0.07	1.23	0.92	-1.77	0.6	-0.07	2.38	-0.19	1.47	1.14	-2.17	-2.25	-1.92	-3.73	
M2	-0.0025	0.2691	3.0628	-336.227	84.8892	-19.5994	5.5491	0.6976	36.4857	28.8465					0.5585
	-0.82	0.82	1.47	-2.24	1.25	-0.91	2.41	0.3	1.86	2.39					
M3	-0.0007	-0.1443	3.251	-484.015	-225.954	-54.5685					-109.811	-355.589	-1925.49	-1578.3	0.4512
	-0.21	-0.36	1.41	-2.05	-2.78	-1.66					-1.91	-4.56	-2.86	-4.35	
Panel B: Engle's (2002) integrated dynamic conditional correlation multivariate GARCH model															
M0	-0.0036	-0.8221	6.1104	-697.973	-220.903	-87.2019									0.3762
	-1.12	-2.23	3.39	-3.32	-3.05	-3.35									
M1	0.0008	-0.0228	1.1453	-210.319	102.804	31.6156	6.1674	-2.7213	-64.5791	57.6871	-120.92	-54.8821	-783.348	-1049.55	0.5729
	0.27	-0.06	0.56	-1.13	1.4	1.33	2.66	-0.96	-1.8	3.27	-2.78	-0.73	-1.48	-2.79	
M2	-0.0004	-0.0505	1.1678	-133.884	143.749	2.0568	6.7178	-2.151	-58.236	71.2528					0.5505
	-0.16	-0.16	0.66	-0.82	1.97	0.1	2.95	-0.84	-1.63	4.05					
M3	-0.0031	-0.1004	4.1541	-242.87	-255.367	-29.9655					-119.676	-427.171	-1993.7	-1749.03	0.4388
	-0.93	-0.26	2.16	-1.26	-3.36	-1.04					-2.28	-5.59	-3.1	-4.78	