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# THE FOREIGN EXCHANGE RATE UNDER RATIONAL EXPECTATION

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## I. INTRODRCTION

In this paper, the trace of the foreign exchange rate under rational expectation is observed along the whole time horizon from very short run to long run with various economic and financial environments, different government policies, and different numbers of stock variables which cause dynamic stock adjustments. For the analysis, we adopt a continuous time model, which can be interpreted as a discrete end−of−period equilibrium model when the length of the time unit goes to zero. The financial sector of the economy is described with several stock equilibrium conditions for financial assets. In the end of period model, asset markets are viewed as a one-period forward markets(Buiter 1981). To those stock equilibrium conditions for financial assets, are added flow equilibrium conditions for the goods market, and a few differential equations for the dynamic evolution of the economy. That kind of stock and flow combination has been widely used in this field. One reason for the popularity of the somewhat awkward combination in the continuous time setting is, of course, its mathematical tractability.

There are already a lot of similar studies on the behavior of the foreign exchange rate. For example, Kouri(1976) derives a behavior of the foreign exchange rate from a world where there is only one good and no interest−bearing financial assets. There, both fiscal and monetary expansion cause the foreign exchange rate to depreciate in the short run. Dornbusch(1976) removes the assumption of the purchasing power parity in his model, and instead introduces interest-bearing financial assets and the assumption of the uncovered interest parity. He expects a short run overshooting depriciation of the exchange rate on a monetary expansion. In another paper of Dornbusch and Fischer (1980), they revive the importance of the role of the current account balance in determining the exchange rate, and predict a short run nonovershooting depreciation of the exchange

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rate on a monetary expansion. All the models are very neat, and will simplified to focus on what the authors attempt to show. Here in this thesis, we choose the opposite direction. We use a very standard textbook model and make a kind of sensitivity analysis for the behavior of the exchange rate.

We attempt to find how much some conclusions of previous papers are valid and robust when we use different model specifications and complications, but maintain a fairly standard model as the common base. In other words, we have a question in mind whether there exist, under different but basically similar model settings, simple or relatively general propositions applicable to the relation between the foreign exchange rate and other economic variables. We remove artificial assumptions step by step on the integration of the international economy such as purchasing power parity and uncovered interest parity, and at the same time attempt to come nearer to a more intuitively appealing economy such as an economy with equity capital and real investment. It is of course not true that a model with a lot more complications and similar appearances to the reality can say more truth and have better prediction power than a simple and tidy model. It seems, however, worthwhile to see how widely different predictions different model specifications can bring about and how robust some conclusions are.

Our concern is a small open economy, and we assume that foreign variables are exogenous and that there is no repercussion from the large world. The economy accepts foreign prices of goods and financial assets as exogenously given. It is, however, assumed that the economy has an autonomy in pricing its own goods when home goods are different from foreign goods. In order to simplify the analysis, it is further assumed that the large world is not interested in the financial debt denominated by the currency of the small country, that is foreigners do not hold domestic currency or bonds. With that simplifying assumption, the net increase in domestic foreign-currency denominated assets is identical with the current account surplus. One of the most important advances in recent economics is the explicit incorporation of economic agents expectations in the model, especially those expectation formation used through this study is rational expectation, and it is called complete perfect foresight in models without explicit treatment of the stochastic properties of the economic variables, i.e, certainty models.

A small open economy controlled by strict purchasing power parity and uncovered interest parity is studied in section 2. Since purchasing power parity is hardly sustainable in the short run by empirical evidence, we discard that restriction in section 3 and study an economy integrated to all the assumptions on ecomomic integration in section 4, and we add equity to the already available financial instruments—money and bonds. In the conclusion of his paper(1977), Niehan says that the most serious deficiency in his paper may be the assumption that export and import values react instantaneously to changes in prices and the exchange rate, while, in reality, this reaction is distributed over a period of years. Fully acknowledging the importance of his suggestion, we attempt to incorporate that property into our model. Since the model in section 4 comes to have such a large number of variables as to make analytical approach futile, we loglinearize an original nonlinear model and attempt numerical solutions. Our findings are summarized in the conclusion.

## II. A SMALL OPEN ECONOMY WITH PURCHASING POWER PARITY(PPP) AND UNCOVERED INTEREST PARITY(UIP)

In this section, we establish a small open economy which is directly connected with the large outside world economy by purchasing power parity and uncovered interest parity.

MODEL (A)

1 $M / P = L(R,Y,W)$

2 $B / P = J(R,Y,W)$

3 $EF/P = H(R,Y,W)$

4 $R = R^* + \varepsilon$

5 $W = M/P + B/P + EF/P$

6 $Y = Y_F$

7 $Y = C(YD,W) + G + NX$

8 $PYD = VY + RB + ER^*F - PT - \pi PW + \varepsilon EF$

9 $E\bar{F} = PNX + ER^*F$

10 $\bar{M} + \bar{B} + PT = VG + RB$

11 $\pi = \bar{P} / P$

12 $\varepsilon = \bar{E} / E$

where

M = nominal money supply

B = domestic government's debt

F = foreign bonds

W = real private wealth

P = domestic consumption price level of the package of home and foreign goods

V = price of home goods

E = foreign exchange rate, or domestic price of foreign goods, assuming foreign currency price of foreign goods is 1

R = nominal interest rate on domestic government bonds

$R^*$ = nominal interest rate on foreign bonds

Y = aggregate output

YF = full employment level of aggregate output

NX = net export, i.e, the excess of exports over imports, in home currency

C = total private absorption of the economy

T = real tax revenue

G = real government expenditure

$\pi$ = expected rate of inflation

$\varepsilon$ = expected rate of depreciation of exchange rate

$\upsilon$ = expected rate of inflation of home goods price

$\bar{P}$ = dP/dt, derivative of comsumer price with respect to time

$\bar{P}/P$ = actual rate of inflation of consumer price = $\phi$

$\bar{V}/V$ = actual rate of inflation of home goods price = v

The economy produces a single composite commodity which is assumed a perfect substitute for the foreign goods. Hence, absolute PPP is implied by the one good world. Equation 1 describes money market equilibrium, equation 2 domestic bond market equilibrium, equation 3 domestic foreign bond market equilibrium. All bonds are assumed to pay variable rate of interest. Equation 5 defines private real wealth. UIP is postulated in equation 4. Domestic interest rate is equal to nominal interest rate plus expected depreciation of home currency. Equation 6 says that domestic output is always equal to full employment output level,which is assumed fixed. Such full employment is guaranteed in our small open economy and one-good model because the amount of home goods left over domestic absorption are always absorbed by the large outside world. The trade account balance of payments, NX, is residually determined by the equation 7, where the economy's absorption is a function of disposable income and wealth. Henceforth, we can understand the residual NX as NX(YD, W, G). Disposable income is defined in equation 8. It is composed of domestic output, interest income on domestic and foreign bonds, taxes and expected capital gains. Equation 9 is the balance of payments identity, and shows current account surplus or capital account deficit, which comprises increments to the existing domestic supply of foreign bonds. Under our assumption that international financial transactions are made with foreign currency, the current account balance of payments is the only way to change the domestic supply of foreign bonds. Equation 10 describes government's budget constraint in nominal terms. Government expenditure and interest payments on its bonds are financed by tax revenue or issuing more money or bonds. Perfect foresight on the rate of inflation and depreciation are defined in equation 11 and 12.

With our assumption on PPP and exogenous foreign price level, we can identify P or V with E without loss of generality by assuming the foreign currency price of foreign goods is unity. Furthermore, UIP permits us to aggregate foreign bonds and domestic bonds. For the tractability of our model, we transform all the nominal terms into real terms, and assume that government maintains the real value of government bonds, i.e., $\bar{b} = 0$. The cost for the transformation is not trivial. We are to deal with stationary rate of depreciation rather than stationary level of the foreign exchange rate. The result of the simplification is model (B) below.

MODEL (B)

1 $m = L(R^* + \varepsilon, W) \quad L_1 < 0, \ L_2 > 0$

2 $b + F = J(R^* + \varepsilon, W) \quad J_1 < 0, \ J_2 > 0$

3 $W = m + b + F$

4 $YD = Y_F + Rb + R^*F - T - e(m + b)$

$\quad = Y_F + R^*(b + F) - T - em$

5 $\bar{F} = NX(YD,W,G) + R^*F$

$NX_1 < 0, \ NX_2 < 0, \ NX_3 = 1$

6 $\bar{m} + em + \bar{b} + eb + T = G + Rb$

where

$L_1 =$ partial derivative of real money balances with respect to domestic interest rate

$L_2 =$ partial derivative of real money balance with respect to wealth

$NX_1$ =partial derivative of net exports with respect to disposable income

$NX_2$ =partial derivative of net exports with respect to wealth

$NX_3$ =partial derivative of net exports with respect to government expenditure

The endogenous variables in the model are e, W, YD, m and F, among which m and F are state variables, which are defined as the smallest set of variables determining the state of the dynamic system. Other endogenous variables are called target variables or short term variables. Exogenous variables in this model are b, Yf, G and $R^*$ Equation 1 and 2 are not independent due to wealth constraint equation 3. This model is basically identical with Kouri's (1976). There are, however, bonds and interest income in this model. Government does not increase money at a constant rate as in Kouri's, but increases money only when it needs to finance its deficit.

### 1. SHORT-RUN ANALYSIS

The depreciation rate of the exchange rate is completely determined in the asset market in this model. There is no feedback effect from the goods market on the rate of depreciation because domestic output is exogenously fixed at its potential level. Let us assume for the time being that the exchange rate moves continuously and smoothly over time at any cases. We shall consider the effect on the short run equilibrium value for the depreciation rate of

(i) open market purchase of domestic bonds

(ii) an increase in the government expenditure

The short run multipliers are derived from the equation below of differential variables.

$$L_1 de = (1-L_4)dm - L_2 db - L_2 df$$

Therefore, the multiplier for the monetary expansion is

$$de/dm \mid dm = -db = -de/db \mid db = -dm = 1/L_1 < 0$$

It is assumed for convenience that the open market operation is implemented in terms of real balances and real bonds. The multiplier for the fiscal expansion is

$$de/dG = 0$$

As the multiplier shows us, the open market purchase lowers the rate of depreciation in the short run. Ceteris paribus. Open market operation raises excess supply of money as much as the amount of the operation itself because there is no change in the wealth and thus in the demand for real balances. In order to restore the money market equilibrium, the demand for money should be raised. This can be accomplished by lower rate of interest, which can happen only when people expect lower rate of depreciation under our assumed world of UIP. Government fiscal expansion does not affect the exchange rate as we expect in the pure asset market approach to the exchange rate.

Those propositions above look fairly counter-intuitive from the viewpoint of an economy which is not connected to the world by UIP. In the case of open market operation, for example, monetary expansion, ceteris paribus, usually brings about excess supply of money, and this causes the rate of interest to be lowered and the price level to be raised. In the world of only long-term PPP and no UIP, the higher price level will accompany depreciation of the exchange rate sooner or later. There is no reason to expect current or future appreciation of the exchange rate as a result of the current monetary expansion.

The story above, however, is not true if the efficiency of the world financial market makes UIP possible. In the world of UIP, lower rate of interest must be connected with expected (more accelerated) appreciation of the exchange rate. Domestic rate of current interest lower than the world level is not compatible with expectation on the depreciation of the exchange rate because their coexistence implies. With the help of the prefect foresight, existence of the unexploited profit opportunities in the financial market, which contradicts the assumption of the efficiency of the market. In short, appreciation is possible when money expands. This is one of the most importnt ideas the monetary approach introduces, although that idea is still far from being supported with satisfactory empirical verifications.

## 2. INTRINSIC DYNAMICS AND STABILITY OF THE ECONOMY

Intrinsic dynamics is defined as the evolution of endogenous variables over time without any changes in the exogenous variables, while extrinsic dynamics as the evolution of endogenous variables caused by changes in the exogenous variables. In the model (B), the intrinsic dynamics is governed by changes in the real balances and foreign assets. The short run equilibrium value of the rate of depriciation of the home currency adjusts every instant to the dynamic evolution of the two assets. An increase in the real balances lowers the depreciation rate.

$$de/dm = e_m = (1 - L_2)/L_1 < 0$$

The multiplier is identical with the previous one for open market operation in the subsection 1 except that it includes an additional term, $-L_2/L_1$, due to the wealth effevt of real balances.

On the other hand, the rate of depreciation is accelerated by increases in the foreign asset postition.

$$de/dF = e_F = -L_2/L_1 > 0$$

More foreign bonds, ceteris paribus, imply more wealth and thus more demand for real balances. The excess demand for real balances should be removed by higher interest rate. That implies accelerated depreciation of the exchange rate under UIP and perfect foresight.

Local stability of the system can be analyzed by linearizing the differential equations of the model in the neighborhood of the long run steady state equilibrium. Let us assume the depreciation rate of the exchange rate is zero at the initial state. Then we obtain the following system of differential equations.

$$\begin{bmatrix} \bar{m} \\ \bar{F} \end{bmatrix} = A \begin{bmatrix} m - \dot{\bar{m}} \\ F - \bar{F} \end{bmatrix}$$

where A is $2 \times 2$ matrix and its elements, assumed time invariant, are as follows ;

$$A_{11} = -e_m \bar{m}$$

$$A_{12} = -e_f \bar{m}$$

$$A_{21} = -NX_1 e_m \bar{m} + NX_2$$

$$A_{22} = NX_1 R^* - NX_1 e_f \bar{m} + NX_2 + R^*$$

Henceforth, we do not use a bar on a notation of an economic variable to express long term value if there is no cause for concern for confusion. The signs of the elements are as follows ;

$$\text{sign}(A_{11}) = +$$

$$\text{sign}(A_{12}) = -$$

$$\text{sign}(A_{21}) = -$$

$$\text{sign}(A_{22}) = ?$$

We can fairly safely assume that the elasticity of money demand with respect to wealth is trivial on the basis of rationality and empirical evidences. Then $A_{22}$ is approximated by $NX_1R^* + NX_2 + R^*$. Since $\bar{Y} - C - G = NX$ in this one good and fixed output model, $NX_1 = -C_1$, partial derivative of consumption with respect to disposable income, $C_2$ with respect to wealth. Therefore, $A_{22} = (1 - C_1)R^* - C_2$. It is very probable that $\text{sign}(A_{22})$ is negative under reasonable range of values $C_1$, $R^*$ and $C_2$ might take, say, $C_1 = 0.6 - 0.8$, $C_2 = 0.05 - 0.1$, and $R^* = 0.02 - 0.1$. Therefore, the matrix A has negative determinant, which implies the economy is unstable and the stationary state is a saddle point.

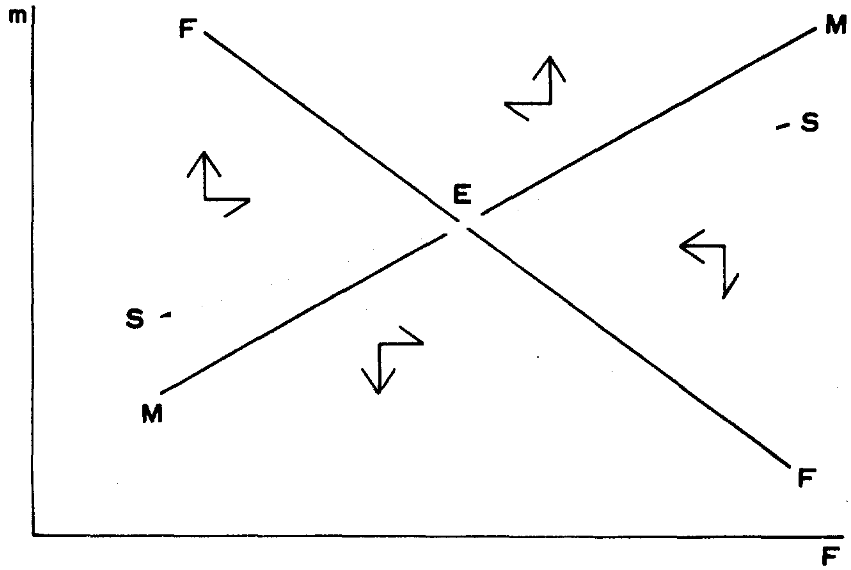


fig. I I-I

We draw three paths in the fig.II − 1 ; MM where real balances are stationary, FF where foreign asset position is stationary and SS which is the oney trajectory leading to the stationary state. The slope of the curve MM is derived from the condition $\bar{m} = 0$ ;

$$-e_m \bar{m}\ dm - e_f \bar{m}\ dF = 0$$

$$dm/dF \mid MM = L_2/(1 - L_2) > 0$$

The wealth elasticity of the real money balaces is believed a priori to be nonnegative. There is, however, no empirical support for its significance, and $L_2$ is often assumed to be so small that a term multiplied by $L_2$ is ignored when we try to figure out a sign of a multiplier which has other conflicting terms with those $L_2$−multiplied terms. When a $L_2$−multiplied term stands alone without other conflicting terms like $dm/dF \mid MM = L_2/(1 - L_2)$ above, we do not ignore $L_2$ despite its size. FF is computed from the stationary condition $\bar{F} = 0$ ;

$$dm/dF \mid FF$$

$$= -(NX_1R^* - NX_1e_f \bar{m} + NX_2 + R^*)/(-NX_1e_m \bar{m} + NX_2) < 0$$

3. LONG RUN STATLONARY STATE EQUILIBRLUM

At stationary state, the stock of real balances and that of foreign bonds stop changing, and the economy ceases to evolve dynamically. The following system of 5 simultaneous equations determines the long run equilibrium valuse for $\bar{e}$, $\bar{m}$, $\bar{F}$, $\bar{W}$, $\bar{Y}D$.

$$\bar{m}\, L(R^* + \bar{e},\ \bar{W})$$
$$\bar{W} = \bar{m} + b + \bar{F}$$
$$O = G + R^*b - \bar{e}\,\bar{m} - T$$
$$O = NX(\bar{Y}D,\ \bar{W},\ G) + R^*\bar{F}$$
$$\bar{Y}D = Y_F + R^*(b + \bar{F}) - T - \bar{e}\,\bar{m}$$

In the following subsection 4, we will consider how those long run values are affected by changes in policy instruments. It should be remembered that comparative static analysis does not make much sense if the economy is unstable. So far we have assumed that the state variables evolve smoothly. From now on, we discard that restriction, and follow the idea of Sargeant and Wallace (1973), permitting real money balances, m, to immediately jump to on the stable path once the jump of m is made possible by the exchange rate moving discretely. Such state variables as permitted to jump are called nonpredetermined variables, or sometimes forward looking variables. The other state variables are called predetermined or backward looking variables, and move smoothly over time. Discontinuous movement of the real money balances might change our previous results on the short run comparative static analysis.

## 4. EXTRINSIC DYNAMICS

In this subsection we shall consider the effect of monetary and fiscal expansions on the long run equilibrium values for the depreciation rate, real balances and foreign asset position. Here monetary expansion is carried by reducing the real stock of bonds, and fiscal expansion by increasing government expenditure.

### 4-1. MONETARY EXPANSION

The comparative statics on the effect of monetary expansion on the depreciation rate, real balances and foreign asset position is summarized by the following multipliers.

$$de/db = R^*/m > 0$$
$$dm/db = -L_1R^*(NX_1R^* + NX_2 + R^*)/|Z| < 0$$
$$dF/db = NX_2(m + L_1R^*)/|Z| \quad >0, \text{ if } -L_1R^*/m > 1$$
$$\qquad\qquad\qquad <0, \text{ if } -L_1R^*/m < 1$$

where $|Z| = m\{(L_2 - 1)(NX_1R^* + NX_2 + R^*) - L_2NX_2\}$,

by assuming that the depreciation rate is zero at the initial state

Monetary expansion results in the appreciation of the exchange rate in the long run. In deriving the multipliers for real balances and foreign asset position, we assume that the response of demand for money to wealth changes is trivial. Real balances are increased in the long run by monetary expansion. Foreign asset position, however, is of ambiguous direction upon monetary expansion. It depends on the elasticity of money demand with respect to the interest rate. Foreign assets increase if the elasticity is less than 1. Assuming the elasticity is less than 1, we draw a phase diagram which shows movement of the economy upon monetary expansion.

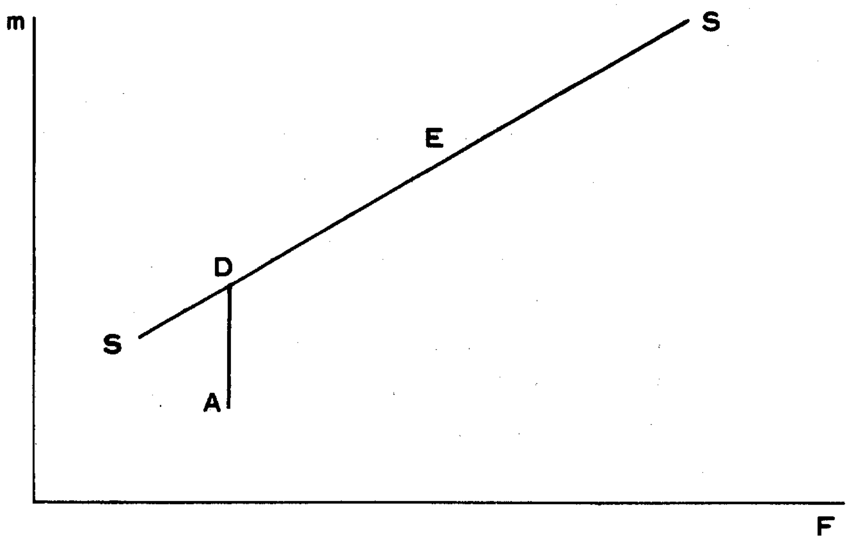


fig. II-2

When there happens an unanticipated money increase, the economy jumps up instantly from the initial equilibrium A to D on the stable arm SS without any change in the foreign asset position. We cannot tell whether or not this initial increase in the real balances is larger than the decrease in the domestic bonds. If the former is larger than the latter in the absolute value, it implies that the foreign exchange rate has discretely appreciated. If the former is smaller than the latter, it implies discrete depreciation of the exchange rate. After the initial upward jump of the real balances, the real balances continuously increase in the adjustment process from D to the long run equilibrium E.

If we assume that the wealth elasticity of real balances is trivial, the long run increase in the real balances is shortened to

$$dm/db = L_1R^*(NX_1R^* + NX_2 + R^*)/|Z|$$
$$= L_1R^*/m$$

With one more assumption that the interest elasticity of real money is less than 1, we can see $dm/d(-b)<1$, that is, the long run increase in the real balances is not greater than the exogenous decrease in the government bonds. This implies that the initial upward jump of the real balances is smaller than the decrease in the bonds, and thus the foreign exchange rate initially depreciates on the shock, because the long run increase in the real balances is larger than the initial discrete movement.

Decreased domestic bonds reduces interest income of the private economy. The upward jump of the real balances increases private wealth, but does not match the initial decrease in the bonds, and thus the private wealth in total shrinks. Just after the initial stabilizing upward jump of the real balances, the economy is put in the situation where its disposable income and wealth ask its residents for more restraints on their consumption than before the shock. The restraints help to accumulate foreign assets through current account surplus.

The rate of depreciation of the exchange rate just after the jump can be approximated by

$$de = e_m\ dm + e_F\ dF = e_b\ db$$
$$= (1 - L_2)dm/L_1 - L_2db/L_1$$

The change in the real balances, dm, is positive due to initial stabilizing jump, and the change in the domestic bonds, db, is negative due to open market purchase of domestic bonds. Therefore, the change in the rate of depreciation, de, is negative.

It is at the same instant as unanticipated monetary expansion occurs that the stabilizing jump of the exchange rate occurs. Suppose that the exchange rate does not jump instantly on the shock, but jumps at another time later onto the stable manifold. This implies that the condition for UIP does not hold at that later moment because the depreciation rate on the jump is infinite. This provides the opportunity for arbitrage profits in that period for investors equipped with perfect foresight. The existence of unexploited arbitrage profit opportunities is in contradiction with the short run equlibrium of the economy.

Let us compare the current case(a) where the exchange rate is permitted to jump for stability with the previous case (b) where it is restricted to move smoothly over time. It is common for both cases that the open market operation brings about excess supply of real balances, and thus requires lower rate of domestic interest for the short run equilibrium in the money market, which in turn implies expectation on appreciation under UIP. The expected rate of appreciation necessary for the short run equilibrium, however, is not identical for both cases. With our assumption that (i) wealth elasticity of real money balances is trivial and (ii)interest elasticity of real money balances is less than unity, the short run increase of the rate of appreciation after exogenous shock is higher in the continuous case (b) than in the initial stabilizing jump case (a). Furthermore, in the case (b), it is almost surely true that the trajectory of the economy does not converge to the stationary state of long run equilibrium.

## 4-2. FISCAL EXPANSION

The long run stationary effects of an increase in government expenditure are given by

$$de/dg > 0$$
$$dm/dg < 0$$
$$dF/dg = L_1NX_2 - NX_1m - m = \ ?$$

The rate of depreciation increases in the long run due to fiscal expansion. Real money balances derease in the long run, but the direction of foreign asset positions is ambiguous.

A phase diagram for unanticipated fiscal expansion, flg.Ⅱ − 3, is drawn below in the similar manner as in subsection 4-1, assuming that

$$dF/dg = L_1NX_2 - NX_1m - m < 0.$$

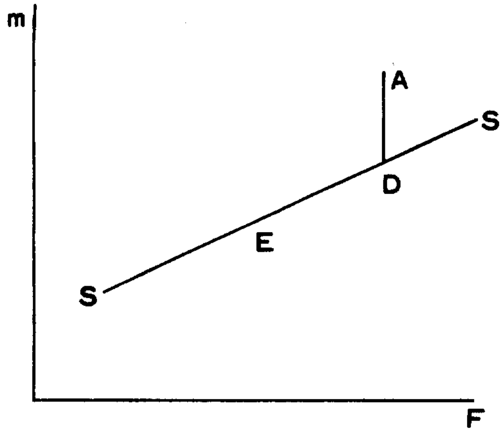


fig. II-3

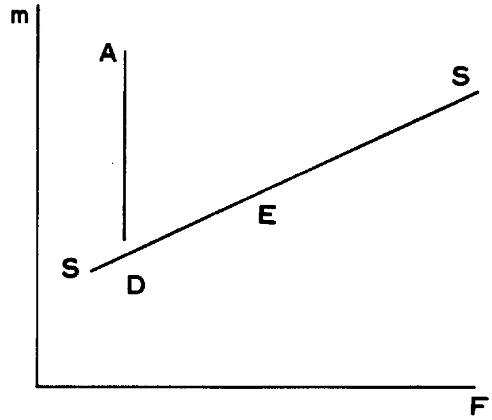


fig. II-4

The exchange rate jumps upward on the shock for the stability of the economy, and the real balances accordingly decrease. The rate of depreciation of the exchange rate just after the jump can be approximated by

$$\begin{aligned} de &= e_m\, dm + e_F\, dF + e_G dG \\ &= (1 - L_2)dm/L_1 \end{aligned}$$

The change in the real balances, dm, is negative due to initial stabilizing depreciation. Government expenditure does not have a direct effect on the rate of depreciation, but has an effect only through changes in the real balances and foreign assets, i.e., $e_G = 0$. Therefore, the change in the rate of depreciation, de, is positive. Real money balances continues to decrease after the initial downward discrete adjustment, and the economy gradually loses its foreign assets.

On the other hand, if we assume $dF/dg > 0$, the phase diagram looks like fig.Ⅱ − 4 above. It is interesting that the immediate effect of the government expenditure on the foreign exchange rate is depreciation regardless of its effect on the long term foreign asset poitions.

## III. A SMALL OPEN ECONOMY WITH UNCOVERED INTEREST RATE PARITY

The hypothesis of purchasing power parity(PPP), especially for short term, is hardly supported by empirical evidence. If we interpret PPP from fairly narrow but not incorrect viewpoint of commodity arbitrage through international trade, the violation of PPP might be due to high transportation and transacion costs, existence of contracts, etc. Since financial market is usually far more efficient than goods market because of lower transaction or information costs, and standardized financial commodities, etc., the hypothesis of uncovered interest rate parity seems less vulnerable than that of purchasing power parity. We discard PPP, but maintain UIP in this section and observe the behavior of the foreign exchange rate. We deal with only the case where the price of the home hoods price is not so flexible as to guarantee full employment at any moment. Henceforth, the price of home goods is sticky in the sense that it does not agilely adjust for market clearance every short term, but has inertia connected to the last period. Economic agents have complete perfect foresight on both the flexible exchange rate and sluggish home goods price.

MODEL (A)

1 $M/P = L(R,Y,W)$

2 $B/P = J(R,Y,W)$

3 $EF/P = H(R,Y,W)$

4 $R = R^* + \varepsilon$

5 $W = M/P + B/P + EF/P$

6 $Y = C(YD,W) + G + NX(YD,W,V/E)$

7 $PYD = VY + RB + ER^*F - PT - \pi PW + \varepsilon EF$

8 $P = V^{\alpha}E^{1-\alpha}$

9 $\bar{V}/V = \theta(Y - \bar{Y})$

10 $E\bar{F} = VNX(YD,W,V/E) + ER^*F$

11 $\bar{M} + \bar{B} + PT = PG + RB$

12 $\pi = \bar{P}/P$

13 $\varepsilon = \bar{E}/E$

14 $\upsilon = \bar{V}/V$

Where all the notations are explained in the previous section except

$\alpha$ = proportion of expenditure spent on home goods

$\upsilon$ = expected rate of inflation of home goods price

$\bar{V}/V$ = actual rate of inflation of home goods price = v

All the equations except 6, 8 and 9 are already explained in the previous section. Equation 6 describes that domestic output is the sum of consumption, government expenditure and net export, all of which are measured in terms of home goods. Since purchasing power parity is no longer assumed to be true, real exchange rate is not constant. Net exports depend on the real exchange rate as well as disposable income and wealth. Here changes in the terms of trade affect real disposable income and wealth, and directly bring about substitution between home goods and foreign goods. Laursen and Metzler effect is easily incorporated in this model because we do not use aggregate output in the absorption function, but disposable income. As is shown in the model (B) below, disposable income is explicitly dependent on the terms of trade. Price index for domestic consumers is defined in equation 8 as a weighted average of the price of home goods and the domestic price of foreign goods. The weight is proportional to the share of expenditure on the goods. Such a weighted price index is theoretically correct if consumers' preference can be described with a homothetic utility function Here it is used just for a convenient and simple approximation to an unknown true price index. The behavior of price for home goods is specified in the equation 9. The price does not flexibly adjust itself in order to achieve a full employment in the economy, but moves slowly in response to a signal of the deviation of current output from the potential full employment output.

Transforming the model (A) in real terms, we obtain model (B) below.

MODEL (B)

1 $m = L(R^* + \varepsilon, Y, W) \qquad L_1 < 0, \quad L_2 > 0, \quad L_3 < 0$

2 $b + xF = J(R^* + \varepsilon, Y, W) \qquad J_1 > 0, \quad J_2 < 0, \quad J_3 < 0$

3 $W = m + b + xF$

4 $Y = C(YD, W) + G + NX(YD, W, \sigma(x))$

$C_1 > 0, \quad C_2 > 0$

$NX_1 < 0, \quad NX_2 < 0, \quad NX_3 < 0$

$1 > C_1 > |NX_1|, \quad 1 > C_2 > |NX_2|$

5 $YD = s(x)\ Y + Rb + xR^*F - T - \pi W + \varepsilon xF$

6 $\pi = \alpha\upsilon + (1-\alpha)\varepsilon$

7 $\upsilon = \theta(Y - Y_F)$

8 $\dot{x} = \alpha(e - v)x$

9 $\bar{F} = \sigma(x) NX(YD, W, \alpha(x)) + R^*F$

10 $\bar{m} + \pi m + \bar{b} + \pi b + T = G + Rb$

where

$x = E/P$

$\alpha(x) = V/E = x^{-1/\alpha}$ = terms of rtade

$\alpha(x)^{-1} = E/V$ = real exchange rate

$s(x) = V/P = X^{(\alpha-1)/\alpha}$

We specify that government maintains the real value of money and bonds, and balance its budget by residually adjusting its tax revenue to its expenditure. With those accommodating fiancial policy and tax policies, the equation 10 is simplified to

$$T = G + (R^* + \varepsilon)b - \pi m - \pi b$$

using this equation of tax revenue, the equation for real disposable income is also rearranged to

$$YD = s(x)\ Y + xR^*F - G + \alpha(\varepsilon - \upsilon)xF$$

It is noteworthy that expected capital gains accrue only to the foreign assets with such government policies. In the model, $\varepsilon, \upsilon$, $\pi$, Y, W, YD, T, x and F are included in the endogenous variables. Exogenous variables are m, b, G, $Y_F$ and $R^*$. It is assumed that x is a forward looking variable, and F is a backward looking variable. Taking the same steps as in the previous section, we find that home currency depreciates in the short run upon monetary expansion, whereas it appreciates upon fiscal expansion. The traces for the endogenous variables are summarized in the table below.

Responses of the economy to the monetary expansion

| endogenous variables | short run | long run |
|---|---|---|
| short term variables | | |
| rate of depreciation | indeterminate | higher rate |
| aggregate output | larger output | larger output |
| state variables | | |
| real exchange rate | higher rate due to step depreciation of home currency | higher rate* |
| foreign asset positions | predetermined | larger position |

Responses of the economy to the fiscal expansion

| endogenous variables | short run | long run |
|---|---|---|
| short term variables | | |
| rate of depreciation | indeterminate | no changes |
| aggregate output | indeterminate | no changes |
| state variables | | |
| real exchange rate | lower rate due to step appreciation of home currency | lower rate* |
| foreign asset positions | predetermined | lower positions |

* A strong assumption on the foreign interest rate, $(C_1 - 1)R^* - NX_2 > 0$, is necessary for the results.

## Ⅳ. AN AUTONOMOUS SMALL OPEN ECONOMY

So far we have excluded equity capital from the financial assets. In this section, we make it available for investmint. At the same time, we finally drop the environmental assumption of uncovered interest parity, and see if it causes a dramatic change in the behavior of the foreign exchange rate. Dispensing with the UIP implies a separate financial equilibrium condition for the foreign currency denominated bonds. However, we assume that equty capital is a perfect substitute for the domestic bonds up to a risk primium, and so, do not postulate a separate equilibrium condition for the equity capital but an arbitrage condition. The main reason to incur such loss of generality is to reduce as many equations as possible which contain very complicated parameters to estimate.

Investment expenditure of accumulation of real capital is taken into account in the models of this chapter, and some of the capital stock is permitted to be idle when the aggregate demand does not reach the supply capacity of the capital.

The government no longer sticks to the balanced budget as in section 3, and finances its deficits by issuing money. We could make the financial behavior of the government more realistic by allowing the government to issue bonds as well for the deficits. However, allowing only money financing makes the model simpler and prevents to invent another arbitrary rule of financing deficits and /or another long term equilibrium condition.

The long term equilibrium is still defined as the stationary state of all the economic variables. We can easily adapt the model such that it has a balanced steady growth in the long run equilibrium. In that case, what makes us unhappy is, among other things, the indefinite growth of foreign assets under our model specification and a possible contradiction to the assumption of small open economy. With our model specification, the balanced steady growth seems to be quite appropriate for a medium term equilibrium, but inconsistent with long term equilibrium. There are so many endogenous variables as to make analytical approach a futile attempt and numerical simulation the only choice.

MODEL (A)

1 $M/P = L(R, R^* = \varepsilon, Y, W)$

2 $B/P + qK = J(R, R^* = \varepsilon, Y, W)$

3 $\beta + \psi(Y)/q = R - \pi + \alpha$

4 $EF/P = J(R, R^* = \varepsilon, Y, W)$

5 $W = M/P + B/P + EF/P + qK$

6 $Y = C(YD, W) + I + G + NX(YD, W, V/E)$

7 $NX(YD, W, V/E) = X(V/E) - (E/V)MX(YD, W, V/E)$

8 $PYD = VY + RB + ER^{*}F - PT + \omega\ cg$

9 $cg = -\pi P\{(M + B + EF)/P\} + \varepsilon P\{EF/P\} \pm \beta P(qK)$

10 $P = V^{\alpha}E^{(1-\alpha)}$

11 $\pi = \alpha\upsilon + (1 - \alpha)\varepsilon$

12 $\upsilon = \theta_{\pi}[Y - F(K)]$

13 $\bar{K} = \theta_{KI}\{\rho Y/(R - \pi + \alpha) - K\}$

14 $\bar{F} = (V/E)\{X(V/E) - (E/V)MN(YD, W, V/E)\} + R^{*}F$

15 $\bar{M} + \bar{B} + PT = VG + RB$

15−1 $\bar{M} = (VG + RB - PT)$

15−2 $\bar{B} = 0$

16 $\upsilon = E(\bar{V}/V) = \bar{V}/V$

17 $\varepsilon = E(\bar{E}/E) = \bar{E}/E$

18 $\beta = E(\bar{q}/q) = \bar{q}/q$

where all the notations are already explained except

$q$ = price of eqiuity capital

$cg$ = capital gains

$\omega$ = proportion of capital gains reflected in the consumption plan

$\beta$ = expected rate of increase of the price of equity capital

$E/V$ = real exchange rate

$V/E$ = terms of trade

Equation 1 to 5 describe the financial sector of the economy. Equation 3 is an arbitrage condition between equity capital and domestic bonds. $\alpha$ denotes a risk primium for holding equity capital, and is assumed to be constant for simplicity. Since foreign bonds are no more perfect substitutes for domestic bonds, a separate equilibrium condition, equation 4, is required for the foreign bonds. Since we will employ numerical approach, we do not have to be so parsimonious with the number of endogenous variables or their relationships. In that regard, we leave intact such nominal varibles as money, bonds and price in the analysis, i.e., we do not convert the model above to a model with real money and bonds as we did in the previous chapters. The numeraire is the weighted combination of home goods as shown in the equation 10. The expected rate of consumer price inflation, $\pi$ in equation 11, needs to be defined with consistency with equation 10 as the weighted average of those of home goods and foreign goods. Equation 6 describes the equilibrium condition for home goods market, and equation 12 the behavior of the home goods price. Equation 7 defines net exports in terms of domestic currency, equation 8 disposable income, and equation 9 capital gains.

Dynamic evolution of the economy is determined by the equations of 13 to 18. Equations 16 to 18 define rational expectation of home goods price, foreign exchange rate and stock price, respectively.

Equation 12 describes capital accumulation. It is identical with investment expenditure, assuming null depreciation. The term $\rho Y/(R-\pi+\alpha)$ represents a desired level of capital with the aggregate output Y and discount rate $R-\pi+\alpha$ ; $\rho$ can be roughly interpreted as an elasticity of the output with respect to the capital. Investment is propelled by the discrepancy of the desired capital stock from the actual stock. The adjustment of the capital stock requires some time lag, and the delay is incorporated in the coefficient $\theta_{K1}$.

There are 11 endogenous variables in the model to solve for. R, e, v, $\beta$, Y are short term variables, and E, V, q, K, F, M are state variables.

We would like to add two more flavors of reality to the model (A) above, although such addition makes the model more arbitrary. The first flavor is related to the equation 13 of capital accumulation. There changes in the real interest rate contributes to the investment expenditure in the same manner as the changes in the aggregate demand by changing the desired level of the capital stock. However, it is generally agreed that changes in the interest rate are not as effective for the investment expenditure as changes in the aggregate demand. We take into account this unbalance in the effectiveness of the two factors by simply adding on more term $\theta_{K2}(Y-F(K))$ to the equation of capital accumulation. The second flavor is connected to the time delay goods price, which seems too important to ignore when we attempt a detailed numerical evolution of the economy from this kind of dynamic models. It is observed that the domestic price of home goods is not as volatile as the foreign exchange rate observed in the financial market. Foreign exporters do not instantaneously adjust the domestic price of their goods to the changes in foreign exchange rate for fear of adverse effect on their profit or market share as well as for the foreign trade convention of using forward market. We introduce the time delay in our model by making the domestic price of foreign goods follow the foreign exchange rate with some time lag. In the long run the change in the foreign exhcange rate is fully reflected in the domestic price of foreign goods.

Taking into account the two flavors of reality, we transform model (A) into model (B) below.

MODEL (B)

1 $M/P = L(R, R^* + \varepsilon, Y, W)$

2 $B/P + qK = J(R, R^* + \varepsilon, Y, W)$

3 $\beta + \psi(Y)/q = R - \pi + \alpha$

4 $EF/P = J(R, R^* + \varepsilon, Y, W)$

5 $W = M/P + B/P + EF/P + qK$

6 $Y = C(YD, W) + I + G + NX(YD, W, VL/EL)$

7 $NX(YD, W, VL/EL) = X(VL/EL) - (EL/VL)MX(YD, W, VL/EL)$

8 $PYD = VLY + RB + ER^*F - PT + \omega\ cg$

9 $cg = -\pi P\{(M + B + EF)/P\} + \varepsilon P\{EF/P\} \pm \beta P(qK)$

10 $P = VL^{\alpha} EL^{(1-\alpha)}$

11 $\pi = \alpha \upsilon 1 + (1-\alpha)\varepsilon 1$

12 $\upsilon = \theta_{Y1}[Y - F(K)]$

13 $\bar{K} = \theta_{K1}\{\rho Y/(R - \pi + \alpha) - K\} + \theta_{K2}(Y - F(K))$

14 $\bar{F} = (VL/E)\{X(VL/EL) - (LE/VL)MN(YD, W, VL/EL)\} + R^{*}F$

15 $\bar{M} + \bar{B} + PT = VLG + RB$

15−1 $\bar{M} = (VLG + RB - PT)$

15−2 $\bar{B} = 0$

16 $\upsilon = E(\bar{V}/V) = \bar{V}/V$

17 $\varepsilon = E(\bar{E}/E) = \bar{E}/E$

18 $\beta = E(\bar{q}/q) = \bar{q}/q$

19 $\bar{V}L/VL = \theta_{VL}(V - VL)$

20 $\bar{E}L/EL = \theta_{EL}(E - EL)$

21 $\upsilon 1 = E(\bar{V}L/VL) = \bar{V}L/VL = \theta_{VL}(V - VL)$

22 $\varepsilon 1 = E(\bar{E}L/EL) = \bar{E}L/EL = \theta_{EL}(E - EL)$

Four notatins, E, EL, V and VL need explanation. Here V is not the price of home goods as in model (A), but is interpreted as an index for the market pressure for the home goods or as a hypothetical price of home goods which might be set if the market were more efficient than it really is. The actual market price is represented by VL which traces V with some time delay. E is the foreign exchange rate applied to the financial transactions. EL is the domestec price of foreign goods (foreign exchange rate multiplied by appropriate foreign currency price of foreign goods) which follows E with some time lag. Equation 19 and 20 express the time delay relationship of home goods price and domestic foreign goods price, respectively. Actually it seems that the invention of VL is not as much necessary as that of EL.

As is shown in the equation 20 and 21, economic agents have rational expectations on the changes in VL and EL as well as those in V and E.

One minor modification is government tax policy in the equation 15−3.
We assume that the total tax is composed of two parts, taxes proportional to the aggregate output (tY) and fixed taxes(TF).

The easiest way to run a numerical simulation on a model is, maybe, to linearize the whole model. It is especially true for the rational expectation model because there is a general analytical solution for the perfect foresight model with time invariant linear system(Buiter 1984). It must be admitted that it is very plausible that the linearized model, especially with time invariant coefficients, can not approximate the true nonlinear model for a large changes in economic variables or for a long time span. We simply assume that the linear relations are not destroyed for the changes in economic variables and time span we are interested in. Model(C) is a linearized version of model (B), and all of the variables except interest rate, R, and foreign bonds, F, are loglinearized (Aoki, 1980). For example, $\hat{M}$ expresses the proportional deviation of M form the initial log term equilibrium value of M, i.e.,

$$\hat{M} = (M - M_0)/M_0,$$

where $M_0$ is the initial equilibrium value

$\delta F$ is the deviation of F from the initial value of F, which is zero :

$$\delta F = F - F_0 = F$$

M′ is the time derivative of $\hat{M}$, i.e.,

$$M' = d(\hat{M})/dt$$

MODEL (C)

1 $(M/P)\hat{M} - (M/P)\hat{P} = L_1\delta R + L_2\delta(R + e) + L_3Y\,\hat{Y} + L_4W\,\hat{W}$

2 $(B/P)\hat{B} - (B/P)\hat{P} + (qK)\hat{q} + (qK)\hat{K} = J_1\delta R + J_2\delta(R^* + e) + J_3Y\,\hat{Y} + J_4W\,\hat{W}$

3 $\delta\beta + (\psi/q)Y\,\hat{Y} - (\psi/q)\hat{q} = \delta R - \delta\pi$

4 $W\hat{W} = (M/P)\hat{M} - (M/P)\hat{P} + (B/P)\hat{B} - (B/P)\hat{P} + (EF/P)(\hat{E} + \hat{F}) - (EF/P)\hat{P} + (qK)\hat{q} + (qK)\hat{K}$

5 $Y\hat{Y} = \{C_1 - (EL/BL)MN_1\}\;YD\;\hat{YD} + \{C_2 - (EL/VL)MN_2\}\;W\;\hat{W}$
$+ \delta_1$
$+ G\,\hat{G}$
$+ \{X' - (EL/VL)MN_3\}\;\{VL/EL)\;\hat{VL} - (VL/EL)\;\hat{E}\}$
$-\; MN\;\{(EL/VL)\;\hat{EL} - (EL/VL)\hat{VL}\}$

6 $P\;YD\;\hat{YD} + P\;YD\;\hat{P} = VL\;Y\;\hat{VL} + B\;\delta R + RB\hat{B}$
$+ ER^*\;F\;\hat{E} + ER^*\;F\hat{F} - PT\hat{P} - PT\hat{T}$
$+ W\;\delta cg$

7 $\delta cg = -\delta\pi(M + B + EF) - \pi(M\hat{M} + B\hat{B} + EF\hat{E} + EF\hat{F})$
$+ EF\delta\varepsilon + \varepsilon EF\hat{E} + \varepsilon EF\hat{E}$
$+ PqK\;\delta\beta + \beta PqK\hat{P} + \beta PqK\hat{q} + \beta PqK\hat{K}$

8 $P\hat{P} = \alpha\;VL^{\alpha}\;EL^{(1-\alpha)}\;\hat{VL} + (1 - \alpha)\;VL^{\alpha}\;EL^{(1-\alpha)}\;\hat{EL}$

9 $\delta\pi = \alpha\;\delta\upsilon 1 + (1\text{-}\alpha)\;\delta\varepsilon 1$

10 $\delta\upsilon 1 = \theta_{VL}(V\hat{V} - VL\;\hat{VL})$

11 $\delta\varepsilon 1 = \theta_{EL}(E\hat{E} - EL\;\hat{EL})$

12 $\delta\upsilon = \theta_{V1}(Y\hat{Y} - F'K\hat{K}) + \theta_{V2}\;\hat{M}^{o}$

13 $K\hat{K}^{o} = \theta_{K1}\;\rho\;Y\hat{Y}/(R - \pi + \alpha)$
$-\;\theta_{K1}\;0.25Y\;(\delta R - \delta\pi)/(R - \pi + \pi)^2$
$-\;\theta_{K1}\;K\hat{K}$

$$+ \theta_{K2}(Y\hat{Y} - F'K\hat{K})$$

14 $\delta\bar{F} = X\{(VL/E)\,\hat{VL} - (VL/E)\,\hat{E}\}$

$$+ (EL/E)\,\{ - MN_1\ YD\ \hat{YD}$$

$$- MN_2\,[(M/P)\,\hat{M} + (B/P)\hat{B} + (EF/P)\,(\hat{E} + \hat{F})$$

$$- (M/P + B/P + EF/P)\,\{\alpha\,\hat{VL} + (1-\alpha)\hat{E}L\}$$

$$+ (qK)\,\hat{q} + (qK)\,\hat{K}]$$

$$+ \{(VL/E)\,X' - MN_3\}\,[(VL/EL)\,\hat{VL} - (VL/EL)\,\hat{E}L]\,\}$$

$$- MN(EL\,\hat{E}L - EL\,\hat{E})/E$$

$$+ R^*\delta F$$

15 $MM\hat{}^{o} = (VLG\hat{VL} + VLG\hat{G} + B\delta R + RB\hat{B} - PT\hat{P} - PT\hat{T})$

16 $\hat{V}^{o} = \delta\upsilon$

17 $\hat{E}^{o} = \delta\varepsilon$

18 $\hat{q}^{o} = \delta\beta$

19 $\hat{VL}^{o} = \theta_{VL}(V\hat{V} - VL\,\hat{VL})$

20 $\hat{E}L^{o} = \theta_{EL}(E\hat{E} - EL\,\hat{E}L)$

In order to make notations simple, we suppress subscript 0 which denote initial value. For example, equation 1

$$(M/P)\,\hat{M} - (M/P)\,\hat{P} = L_1\delta r + L_2\delta(R^* + e) + L_3\,Y\hat{Y} + L_4W\hat{W}$$

is a abbreviated form of

$$(M_0/P_0)\hat{M} - (M_0/P_0)\hat{P} = L_1\delta R + L_2\delta(R^* + e) + L_3Y_0\hat{Y} + L_4W_0\hat{W}$$

The initial values and elasticities used for the time invariant coefficients are given below. Most of the values are chosen from previous studies on numerical simulations(Smith 1980, Nguyen & Turnovsky 1983). Some values are made purely by our intuition on a plausible small open economy.

INITIAL VALUES OF ENDOGENOUS VARIABLES

$M = 100$, $B = 500$, $K = 4000$, $F = 0$, $W = 4600$

$P = 1$, $q = 1$, $E = 1$, $V = 1$

$Y = 1000$, $I = 0$, $YD = 775$, $C = 775$

$X = 200$, $MN = 200$

$R = 0.05$

$e = 0$, $v = 0$, $\beta = 0$, $\pi = 0$

VALUES OF PARAMETERS OR EXOGENOUS VARIABLES

$R^* = 0.05$, $\omega = 0.1$, $\alpha = 0.8$, $t = 0.1$, $G = 225$, $\psi = 0.0625$, $\alpha = 0.0225$

$\theta_{Y1} = 0.0005$, $\theta_{Y2} = 0$, $\theta_{K1} = 0.01$, $\theta_{K2} = 0.1$

Other time-invariant coefficients can be obtained with the information of the inital values above and assumed elasticities or partial derivatives below.

elasticity of money with respect to the rate of interest on the domestic bonds($\eta m_r$)············ 0.3

elasticity of money with respect to the rate of interest on the foreign bonds($\eta m_{rf}$)············ $-0.3$

elasticity of money with respect to income($\eta m_Y$)········· 0.9

elasticity of money with respect to wealth($\eta m_W$)······ 0.1

elasticity of domestic assets with respect to the rate of interest on the domestic bonds($\eta a_r$)············ 0.02

elasticity of domestic assets with respect to the rate of interest on the foreign bonds($\eta a_{rf}$)········· $-0.02$

elasticity of domestic assets with respect to income($\eta a_Y$)······ 0.02

elasticity of domestic assets with respect to wealth($\eta a_W$)··· 1.02

Elasticities of foreign bonds with respect to its argument ($\eta f_r, \eta f_{rf}$, $\eta f_Y$, and $\eta f_W$) can be assessed in the following manner, once we know the other two elasticities.

$$\eta m_r\ W_m + \eta a_r\ W_a + \eta f_r\ W_f = 0$$
$$\eta m_{rf}\ W_m + \eta a_{rf}\ W_a + \eta f_{rf}\ W_f = 0$$
$$\eta m_Y\ W_m + \eta a_Y\ W_a + \eta f_Y\ W_f = 0$$
$$\eta m_W\ W_m + \eta a_W\ W_a + \eta f_W\ W_f = 1$$

where $W_m$, $W_a$ and $W_f$ are the proportion to total wealth of money, domestic assets and foreign bonds, respectively. In this model, we have similar but a little more primitive relationships between the changes in the financial assets, because we assume that the initial level of foreign bonds is zero. For example, $(\partial m / \partial r)dr + (\partial a / \partial r)dr + (\partial f / \partial r)dr = 0$. The values of $L_1$ to $L_4$ and $J_1$ to $J_4$ are easily derived form the above information of elasticities. For example, $L_1 = \eta m_r(M / P) / r$.

$C_1 = 0.75$, $C_2 = 0.05$

$MN_1 = 0.15$, $MN_2 = 0.01$

$MN_1$ and $MN_2$ are obtained by solving $MN_1. + C_1 = MN_2 / C_2 = 1 - \alpha$, where $1 - \alpha$ is the proportion of expenditure spent to foreign goods.

$MN_3 = -140$, $X' = 70$

As long as the sum of $|MN_3|$ and X′ is greater than 200, the depreciation of home currency can, ceteris paribus, improve the trade balance.

$\psi' = 0.00004$

$F' = 0.0625$

The analytical solution to the problem (QA) of rational expectation below is given by (SA) with Buiter's notations(1984).

(QA)

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = A \begin{bmatrix} x \\ y \end{bmatrix} + BZ$$

(SA1) Assuming there are as many predetermined variables as stalbe roots, the trajectories of x and y are given by

$$y(t) = W_{21}W_{11}^{-1}x(t) - V_{22}^{-1}\,{}_t\!\int^{\infty} \exp(\Lambda_2(t-\tau))Dz(\tau)d\tau$$

$$\begin{aligned} x(t) = {} & W_{11}\exp(\Lambda_1(t-t_o))W_{11}^{-1}x_o(t_o) \\ & + {}_{t_O}\!\int^{t} W_{11}\exp(\Lambda_1(t-s))W_{11}^{-1}B_1z(s)ds \\ & - {}_{t_O}\!\int^{t} W_{11}\exp(\Lambda_1(t-s))\{\Lambda_1V_{12}V_{22}^{-1} + W_{11}^{-1}W_{12}\Lambda_2\} \\ & \qquad * [\,{}_s\!\int^{\infty} \exp(\Lambda_2(s-\tau))Dz(\tau)d\tau]\, ds \end{aligned}$$

where

$x = n_1$ vector of predetermined variables

$y = n - n_1$ vector of non-predetermined variables

$z = k$ vector of exogenous variables

$A = V^{-1}\Lambda V = W\Lambda W^{-1}$

$V =$ left eigenvectors of A

$W =$ right eigenvectors of A

$\Lambda =$ diagonal matirix with eigenvalues diagonal elements

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

$$V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}$$

$$W = V^{-1}\begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix}$$

$\Lambda_1 = n_1$ by $n_1$ matrix containing the stable roots of A

$\Lambda_2 = (n - n_1)$ by $(n - n_1)$ matrix containing the stable roots of A

$D = V_{21}B_1 + V_{22}B_2$

In the special case when Z is constant for all t in the future, y(t) can be expressed as

$y(t) = W_{21}W_{11}^{-1}\{x(t) - \bar{x}\} + \bar{y}$

where $\bar{x}$ and $\bar{y}$ are the steady state values of x and y respcetively.

(SA2) If there are fewer predtermined variables than stable roots, a unique convergent solution still can be devised by adding proper number of boundary conditions in the form of linear restrictions. When we apply the solution to our model (C), the vectors x, y and z are given by

x = [V K F M EL VL]
y = [E q]
z = [G B]

For the convenience of computation, we use as shocks only permanent constant changes in the exogenous variables or state variables. Fiscal expansion is implemented by 1 % increase in the government expenditure, and monetary expansion by 1 % decrease in the existing bond stock and 5% increase in the existing money stock. Since Monetay expansion is sometimes referred to without connection to the changes in the bond supply, we will also observe the case of 5% increase in money and no changes in bonds.

Our continuous time model needs to be converted to a discrete one for the numerical simulation. The time interval for the purpose is set to 0.1 which gives us the same simulation output as finer intervals. With the given initial values and parameters, the eigenvalues of the economy are given by

[ −0.6 + 0.77i, −0.6−0.77i, 0.21 + 0.1i, 0.21 − 0.1i,
−0.27 + 0.08i, −0.27−0,08i, −0.14, −0.01 ],

which implies that the economy is not stable, but can reach equilibrium through a unique saddle path under our assumption that there are only two nonpredetermined state variables, foreign exchange rate and stock price.

| | r | Y | M | K | F | V | E | q |
|---|---|---|---|---|---|---|---|---|
| 0 | 7% | 6.3% | 0 | 0 | 0 | 0 | 14.9% | 5.8% |
| 0.5 | 1.2% | 8.3% | −.5% | .3% | −5.2 | 1.9% | 15.1% | 5.2% |
| 1 | 1.6% | 8.5% | −.9% | .7% | −10.3 | 4.0% | 15.3% | 4.2% |
| 2 | 1.8% | 6.2% | .6% | 1.4% | −19.4 | 7.6% | 15.5% | 2.2% |
| 3 | 1.5% | 3.3% | 4.3% | 1.9% | −25.9 | 9.7% | 15.5% | 0.9% |
| ∞ | −.0% | .1% | 8.7% | .5% | .2 | 8.6% | 8.4% | .3% |

Table Ⅳ −1 Fiscal Expansion ; Reference Path

The state of the economy is observed at the instant of the shock, a half year, one year, two years and tree years after the shock, and finally at the long run equilibrium. For fiscal expansion, the time path of endogenous variables are given in the table below. Henceforth, we call this path the reference path.

As can be seen in the row of time 0 in the table above, fiscal expansion accompanies a large initial rise in the foreign exchange rate (depreciation of home currency) and stock price. Furthermore, the exchange rate overshoots its long run equilibrium level. It needs to be compared with the analytical results of a fiscal expansion in section 3, where we derive a jump appreciation of the exchange rate on the basis of different environmental assumptions. Aggregate output increases for some time, and its cost is current account deficit and inflation. Interest rate rises in the short run. Eventually, the economic variables which the fiscal policy can significantly influence are nominal money supply and prices, not real income.

If we do not invoke transversality condition and thus do not rely on the initial stabilizing jumps of the exchange rate and stock price, the short run behavior of the economy below turns out quite different from the reference path. Especially, the foreign exchange rate contiues to appreciates despite current account deficit. Since all the state variables are restricted to move smoothly, the response of the economy diverges in the long run.

| | r | Y | M | K | F | V | E | q |
|---|---|---|---|---|---|---|---|---|
| 0 | 0.1% | .6% | 0 | 0 | 0 | 0 | 0 | 0 |
| 0.5 | 0.1% | 1.1% | .8% | .0% | −.6 | .2% | .0% | −.1% |
| 1 | .1% | 1.5% | 1.4% | .1% | −1.6 | .6% | −.0% | −.2% |
| 2 | .1% | 1.9% | 2.7% | .3% | −5.0 | 1.4% | −.1% | −.9% |
| 3 | .1% | 1.8% | 4.3% | .6% | −9.9 | 2.3% | −.4% | −1.9% |
| ∞ | — | — | — | — | — | — | — | — |

Table Ⅳ−2 Fiscal Expansion ; Without Jumps of E and q

It would be useful to compare table Ⅳ−1 with table Ⅳ−3 below which shows the time path of the economy when we do not introduce arbitrary time delay in the domestic price of foreign goods. In this rational expectation model, we cannot find a sharp contrast between the two simulation except that in table Ⅳ−3 the foreign exchange rate does not overshoot its long run equilibrium, and the size of the current account deficit is smaller than the reference path.

| | r | Y | M | K | F | V | E | q |
|---|---|---|---|---|---|---|---|---|
| 0 | .9% | 7.5% | 0 | 0 | 0 | 0 | 5.1% | 2.5% |
| 0.5 | .7% | 3.9% | −.8% | .3% | −.6 | 1.5% | 5.3% | 1.7% |
| 1 | .7% | 2.6% | −.6% | .4% | −.5 | 2.1% | 5.5% | 1.4% |
| 2 | .6% | 1.9% | .4% | .6% | −.2 | 2.7% | 5.8% | 1.1% |
| 3 | .5% | 1.6% | 1.4% | .7% | −.2 | 3.1% | 6.0% | .9% |
| ∞ | −.0% | .1% | 8.7% | .5% | .2 | 8.5% | 8.4% | .3% |

Table Ⅳ−3 Fiscal Expansion : Without Time Delay in EL and VL

Below is the time path of the economy for monetary expansion. The first table is when money is increased by open market operation. The second table shows the case when money is distributed free from a helicopter. As is shown, there are not much differences between the two monetary expansions at least in the short run. The foreign exchange rate overshoots the long run equilibrium, and the economy experiences current account deficit for some time.

| | r | Y | M | K | F | V | E | q |
|---|---|---|---|---|---|---|---|---|
| 0 | −.3% | 2.5% | 5% | 0 | 0 | 0 | 4.1% | 2.3% |
| 0.5 | 1.1% | 3.4% | 3.1% | .2% | −2.0 | .8% | 4.1% | 1.8% |
| 1 | .4% | 3.4% | 1.8% | .4% | −4.3 | 1.6% | 4.1% | 1.3% |
| 2 | .5% | 1.9% | 1.2% | .7% | −8.9 | 2.9% | 4.0% | 2.3% |
| 3 | .5% | .4% | 2.2% | .9% | −12.3 | 3.4% | 3.8% | −.0% |
| ∞ | .0% | .0% | −.1% | .0% | .0 | −.1% | −.1% | .0% |

Table Ⅳ−4 Monetary Expansion : Instantaneous Open Market Operation

| | r | Y | M | K | F | V | E | q |
|---|---|---|---|---|---|---|---|---|
| 0 | −.3% | 2.6% | 5% | 0 | 0 | 0 | 4.1% | 2.3% |
| 0.5 | .1% | 3.6% | 3.2% | .2% | −2.1 | .8% | 4.1% | 1.8% |
| 1 | .4% | 3.6% | 1.9% | .4% | −4.6 | 1.7% | 4.1% | 1.2% |
| 2 | .6% | 2.2% | 1.5% | .7% | −9.8 | 3.1% | 4.0% | .1% |
| 3 | .5% | .7% | 2.8% | .9% | −13.9 | 3.7% | 3.8% | −.1% |
| ∞ | .0% | .0% | .0% | .0% | .0 | .0% | .0% | .0% |

Table Ⅳ−5 Monetary Expansion : Helicopter Scattering

It is observed that the output of the numerical simulation is not very robust to the fairly disputable parameter values like $\theta_{Y1}$, $\theta_{K1}$, $\theta_{K2}$, etc. This observation might suggest that our model specification is not very good because it cannot avoid using such disputable parameters. On the other hand, however, if we accept that our model is not unreasonable and the real economy hardly diverges indefinitely in the long run, the numerical simulation might help us to guess reasonable range of the coefficients.

## V. CONCLUSION

There certainly exists a robust and clear direction in the short run response of the aggregate output to the government policies. However, it is not possible to find such a clear direction in the response of the foreign exchange rate in either long run or short run, although our study gives a general impression that the short run depreciation of the foreign exchange rate to the monetary expansion is relatively robust, compared with the response of the exchange rate to the fiscal expansion. In section 3, fiscal expansion accompanies short run appreciation of the foreign exchange rate under rational expectation, but in the numerical simulation and section 2, it depreciates home currency in any expectation formation. It seems that we still do not have a useful rule-of-thumb which is fairly widely applicalble to the behavior of the foreign exchange rate in response to chanqes in more basic variables. That implies that before an economist say anything about the future path of the exchanqe rate, he needs to carefully check more fundamental factors ; (i) the degree to which the economy is integrated to the world (ii) the sturcture of the goods and financial markets which affect the process of the expectation formation as well as the speed of adjustment of prices of goods and financial assets (iii) the characteristics of the exogenous shocks and the way how they are implemented.

No studies including ours, of course, could be entitled to refute the fundamental usefulness of the simplified models. Our purpose is just to indicate or confirm the possible limitations of the predictions of simplified models. Any models might be incomplete or misleading which attempt to explain the behavior of the foreign exchange rate, but do not consider the role of the foreign exchange rate in equilibrating the current account balance and economic agents expectations on its equilibrium level. However short a period we deal with, those expectations interfere in the market and seems to play an important role. In brief, it is our belief that, in most cases, the role of the foreign exchange rate as the price of foreign goods should not be completely ignored even in those models whose primary concern is short run. All of our models postulate that the current account should be balance eventually. As the result, the short run behavior of the exchange rate is affected by the expectation on the long run balance and may well be a blend of the traditional flow view and modern asset view.

In our numerical simulation in section 4, we focus more on the time paths for fiscal expansion rather than monetary expansion, mainly because our definition of monetary expansion, open market operation, is not so popular in current literature as helicopter scattering. There, as long as we maintain the rational expectation, the initial overshooting is easily observed upon fiscal expansion even in a fairly autonomous economy which dispenses with PPP and UIP. In addition, the initial overshooting is not reduced over time, but augmented for some time before it eventually vanishes. It might be noteworthy that, if we do not restrict the overshooting to the inital overshooting, but include the interim overshooting as well, any dynamic system with higher than first order could experience overshooting. Finally, introducing time delay in the adjustment of the foreign goods price to the foreign exchange rate does not make much difference in the case of rational expectation.

Once we add to our certainty framework stochastic properties on the behavior of the foreign exchange rate and / or expectation errors, it would be even harder to predict the foreign exchange rate. Bubbles in asset prices or asset price changes upon arbitrary and self-fulfilling expectation of them can mostly happen to those assets whose fundamental values are not easy to assess. Although the market fundamentals of foreign currencies are also not easy to assess, especially in the short run, it is hard to believe that bubbles and their bursting are mainly responsible for the most medium term fluctuations of the foreign exchange rate. In that sense, what is mainly responsible for the nontrivial fluctuation or volatility of the foreign exchange rate should be ascribed to the changes in the perception or expectation of the market fundamentals. In reality, the information usually arrives in the market in lump sum, and the market corrects its expectations with the help of the less noisy information as its expectation comes closer to the date of its realization. Furthermore, the foreign exchange market is highly vulnerable to noises because international political changes as well as domestic ones greatly influence the expectaion on the economic fundamentals relevant for the foreign exchange rate. Such market imperfections and noises are of help in understanding (but, not explanining ex ante) abrupt changes in economic variables, especially prices in financial markets, which we often experience even without any corresponding abrupt changes in the current actual state or trend of the economy or government policies.

One of the most disputable but appealing characteristics rational expectation models possess, but other models do not share is the idea that the economy actively searches for its stability in the long run. The economy seems to have been too stable if all the continuous and / or abrupt movements of economic variables are nothing but aimless responses to surprises or stimuli. That speculation might be connected to the Lucas's critique (1976) that the parameters are contingent on the policies, and the behavior of the economy should be derived from more reliable rules of optimization on the part of economic agents. His idea has already widely influenced this field, and seems worthwhile to pursue further although we never expect the application of his idea in the real world goes without serious difficulties and arbitrariness. Anyway, combining economic agents' expectations and the inclination for stability of the economy with other market fundamentals would make us well equipped to explain the behavior of the foreign exchange rate. However, it should be remembered that, in our analytical models, one of the most intricate factors which makes the short run behavior of foreign exchange rate unpredictable is very existence of the expectations, because it increases the number of the endogenous variables beyond the level that an analytical model can comfortably handle.

It must be admitted that our methodology and model specifications also have a lot of limitations and peculiarities. There are a few serious restrictions in applying our model generally to the real world. One of the most serious restrictions might be caused by our assumption on the external capital flows. We have assumed that the outside world is not interested in the home currency denominated assets, and thus formulate a nice simple motion of law for the foreign assets. Foreigners do not want to hold domestic currency denominated assets at all. As the result, the history of the balance in the current account is directly connected to the evolution of the foreign assets, and external financial transactions independent of the real sector are completely ignored. This story might be the case only for the really and truly small economy.

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# THE FOREIGN EXCHANGE RATE UNDER RATIONAL EXPECTATION

Ilseong Yu

SUMMARY

By using deterministic dynamic models, we observe the behavior of the foreign exchange rate of a small open economy with rational expectation formation and different restrictions on the international economic integrations. First, an economy connected to the world by purchasing power parity and uncovered interest parity is studied in the next section. In both sections, financial assets available in the economy are domestic money and bonds. Stocks are added as a financial instrument in the next section, and real capital accumulation is also taken into account. Furthermore, the economy concerned there is fairly autonomous, and not directly governed by either purchasing power parity or uncovered interest parity. The expectation formation used throughout the whole paper is complete perfect foresight, which is the certainty version of rational expectation and free from any forecast errors.

It is found that upon monetary expansion the short run depreciation of the foreign exchange rate is a fairly robust result regardless of the degree of the international economic integration, while it is not true for fiscal expansion.

The expectation on the long run state significantly affects the short run response of the exchange rate. All of our models postulate that the current account should be balanced eventually. As the result, the short run behavior of the exchange rate is affected by the expectation on the long run balance and may well be a blend of the traditional flow view and modern asset view.

The initial overshooting of the exchange rate is easily observed even in the fairly autonomous economy. Furthermore, the initial overshooting is not reduced over time, but augmented for some time before it is eventually eliminated. As long as we maintain rational expectaion, introducing time delay in the adjustment of the foreign goods price to the foreign exchange rate does not make much difference.

# 理性的 期待下의 換率行態分析

柳日盛

국문초록

본 연구에서는 변동환율을 취하고 있는 small open economy 가 대외경제에 얼마나 긴밀하게 연결되어 있는가에 따라 그 경제의 환율이 어떻게 달리 반응하는가를 살폈다. 그 경제주체들은 대단히 효율적으로 필요한 정보를 활용해서 미래를 예측한다. 즉 이성적 기대(rational expectation) 를 갖는다고 가정한다. Small open economy의 환율을 분석한 기존의 모델들 대부분이 어떤 특정한 국제경제여건에 국한하여서 가능하면 단순한 모델들을 설정했기 때문에 그 결론들이 얼마나 광범위하게 다른 경우에도 유효한지 쉽게 알 수가 없다. 본 연구에선 점차적으로 현실적인 요소들을 모델에 가미해가고, 현대의 고도정보처리능력에 가장 적합한 기대형태인 이성적 기대를 다룸으로써 변동환율의 움직임에 어떤 일반성이 있을 수 있는가를 공부했다.

구체적으로, 제2장에는 국내경제가 국제경제와 구매력동등가설 (purchasing power parity)과 이자율동등가성(uncovered interest parity)로 긴밀히 연결되어 있는 경우를 살피고, 제3장에는 구매력동등가설은 적용되지 않고 이자율동등가설만 국내경제에 유효한 경우를 살폈다. 2장과 3장에서 국내투자자가 투자할 수 있는 금융자산은 통화와 국내.외 채권이며 주식시장은 고려되지 않았다. 4장에서는 상당히 자립적이고 현실적인 요소가 많이 반영된 경제를 분석하였다. 즉 국내경제가 국제경제와 구매동등가설이나 이자율동등가설로 직접적으로 연결되지 아니한 상황에서 주식을 포함한 모든 금융자산이 투자대상자산으로 가능하고, 또 환율의 변동이 재화의 국내가격에 반영되는데 어느 정도의 시차가 요구되는 경우를 살펴 보았다. 여기서는 내재변수의 수가 많은 관계로 numerical simulation을 이용했다.

본 연구의 결론 일부로서 첫째, 자국경제의 통화가 팽창되는 경우, 그 경제의 국제경제유착정도에 상관없이 자국통화의 평가절하를 곧 유발하였다. 재정팽창의 경우에는 통화팽창의 경우처럼 환율의 방향에 대한 일반적인 결론을 얻지 못 했다. 둘째, 환율의 움직임에 대해 최근의 자산가격 모델(asset model)들은 과러 전통적인 Keynesian모델들과는 다른 설명을 하고 있는데 본 연구에서는 단기적으로 금융시장에서 자산의 수급일치가 균형조건으로 고려되고, 장기적으로는 경상수지일치가 균형조건으로 포함되었다. 그 결과 장기균형을 예측하는 경제주체들의 기대가 현재환율의 움직임에 큰 영향력을 미침으로써, 전통적인 Keynesian모델들의 단기예측 유효성을 무시할 수 없음을 보였다. 세째, 개방된 경제에서 변동환율의 초기과민반응(overshooting)이 그것이 미칠 수 있는 왜곡된 signal효과 등으로 인해 상당히 염려. 논의되고 있는데, 본 연구 4장의 경제는 상당히 자립적이고 자국통화로 표시된 채권이 국제적으로 수용되지 않음에도 불구하고, 경제주체들이 이성적기대를 견지하는 한, 환율의 초기과민반응은 쉽게 관찰될 수 있었다. 네째, 환율의 변동이 재화의 국내통화가격에 반영되는데 시차를 인정한 경우, 경제주체들이 이성적기대를 갖는한, 시차도입 후 뚜렷이 다른 경제양상을 보이지 않았다.